Coherent backscattering by polydisperse discrete random media: exact T-matrix results

Michael I. Mishchenko,[1,*] Janna M. Dlugach,[2] and Daniel W. Mackowski[3]

[1]*NASA Goddard Institute for Space Studies, 2880 Broadway, New York, New York 10025, USA*

[2]*Main Astronomical Observatory of the National Academy of Sciences of Ukraine, 27 Zabolotny Street, 03680, Kyiv, Ukraine*

[3]*Department of Mechanical Engineering, Auburn University, Auburn, Alabama 36849, USA*

**Corresponding author: michael.i.mishchenko@nasa.gov*

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The numerically exact superposition T-matrix method is used to compute, for the first time to our knowledge, electromagnetic scattering by finite spherical volumes composed of polydisperse mixtures of spherical particles with different size parameters or different refractive indices. The backscattering patterns calculated in the far-field zone of the polydisperse multiparticle volumes reveal unequivocally the classical manifestations of the effect of weak localization of electromagnetic waves in discrete random media, thereby corroborating the universal interference nature of coherent backscattering. The polarization opposition effect is shown to be the least robust manifestation of weak localization fading away with increasing particle size parameter.

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Coherent backscattering (CB) or weak localization of electromagnetic waves in discrete random media is a remarkable phenomenon [1–4] explained qualitatively in the framework of far-field Foldy–Lax equations as constructive interference of partial wavelets caused by "conjugate" multiparticle sequences [5]. The quantitative corroboration of this qualitative explanation based on direct computer solutions of the Maxwell equations is a nontrivial problem that has been the subject of active recent research [6–11]. Although CB has been viewed as a ubiquitous phenomenon [1–4], expected to exist for polydisperse as well as monodisperse particulate media [5], numerically exact modeling results published so far have been obtained for monodisperse particles only. It is, therefore, important to verify the qualitative explanation of CB and demonstrate its universal physical character by means of numerically exact computer calculations for polydisperse discrete random media.

We address this problem using a direct computer solver of the Maxwell equations for a multisphere group called the superposition T-matrix method (STMM) [12]. Within the range of numerical convergence, the corresponding computer program [13,14] generates results with a guaranteed number of accurate decimals, which makes STMM a numerically exact technique. Our model of discrete random medium is an imaginary spherical volume randomly filled with N nonoverlapping spherical particles (Fig. 1). The dimension of the volume is defined by its size parameter kR, where k is the wavenumber in the homogeneous space surrounding the particles and R is the volume radius. The particle size parameter is given by kr, r being the particle radius.

To model statistical randomness of particle positions within the imaginary spherical volume, we use one randomly configured N-particle group and average all optical observables over the uniform orientation distribution of this configuration with respect to the laboratory reference frame. This procedure yields an infinite continuous set of realizations of the scattering volume and allows us to use the highly efficient STMM orientation averaging procedure [12].

The statistically random particulate volume is illuminated by a plane electromagnetic wave or a parallel quasi-monochromatic beam of light propagating in the incidence direction $\hat{\mathbf{n}}^{\rm inc}$ (Fig. 1). The observation point is located in the far-field zone of the entire volume in the scattering direction $\hat{\mathbf{n}}^{\rm sca}$. Because the scattering properties of the particulate volume are averaged over all orientations of an N-particle group, they depend only on the scattering angle Θ provided that the Stokes parameters of the incident and scattered light are defined relative to the scattering plane. The transformation of the Stokes parameters I, Q, U, and V upon the far-field scattering is described by the normalized Stokes scattering matrix of the entire volume [15]:

$$\begin{bmatrix} I^{\rm sca} \\ Q^{\rm sca} \\ U^{\rm sca} \\ V^{\rm sca} \end{bmatrix} \propto \begin{bmatrix} a_1(\Theta) & b_1(\Theta) & 0 & 0 \\ b_1(\Theta) & a_2(\Theta) & 0 & 0 \\ 0 & 0 & a_3(\Theta) & b_2(\Theta) \\ 0 & 0 & -b_2(\Theta) & a_4(\Theta) \end{bmatrix} \begin{bmatrix} I^{\rm inc} \\ Q^{\rm inc} \\ U^{\rm inc} \\ V^{\rm inc} \end{bmatrix}. \tag{1}$$

The zeros denote scattering matrix elements negligibly small (in the absolute sense) relative to the other elements at the same scattering angle.

The elements of the scattering matrix are used to define specific optical observables corresponding to

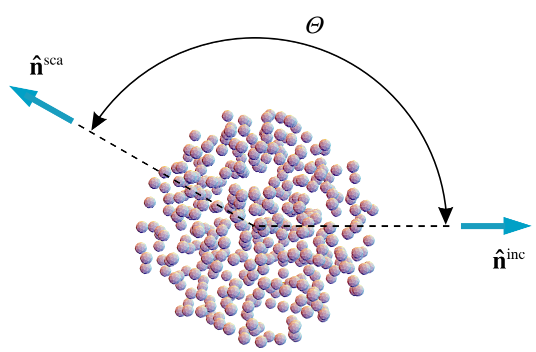

Fig. 1. (Color online) Electromagnetic scattering by a spherical volume of discrete random medium.

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different types of polarization state of the incoming light. For example, if the incident light is unpolarized, then the phase function $a_1(\Theta)$ describes the angular distribution of the scattered intensity, while the ratio $-b_1(\Theta)/a_1(\Theta)$ is the corresponding degree of linear polarization. If the incident radiation is polarized linearly in the scattering plane, then the angular distribution of the cross-polarized scattered intensity is given by $\frac{1}{2}(I^{\mathrm{sca}} - Q^{\mathrm{sca}}) \propto \frac{1}{2}[a_1(\Theta) - a_2(\Theta)]$ [7].

We model particle polydispersity by mixing within one volume particles with different size parameters or different refractive indices. In the former instance, we first use the procedure described in [16] to generate random positions of $N = 400$ particles with a size parameter of $kr = 3$ inside a $kR = 40$ spherical volume. Then a certain number of these particles are replaced randomly with $kr = 2$ particles having the same refractive index ($m = 1.31$). The centers of the smaller particles coincide with those of the removed larger particles. In the latter instance, we use the same procedure to generate random positions of 240 particles with a size parameter $kr = 4$ and refractive index $m = 1.31$ inside a $kR = 40$ spherical volume. Then a certain number of these particles are replaced randomly with equally sized $m = 1.5$ particles.

The results of the corresponding T-matrix computations are displayed in Figs. 2(a) and 2(b). We depict the backscattering profiles of the degree of linear polarization, as well as of the phase function and the cross-polarized intensity normalized by their respective values at $\Theta = 180°$. The normalized phase functions exhibit narrow backscattering peaks, which are totally absent in the corresponding single-particle phase functions [solid gray and dashed–double-dotted yellow curves in the left-hand panel of Fig. 2(a)] and have an angular semiwidth at

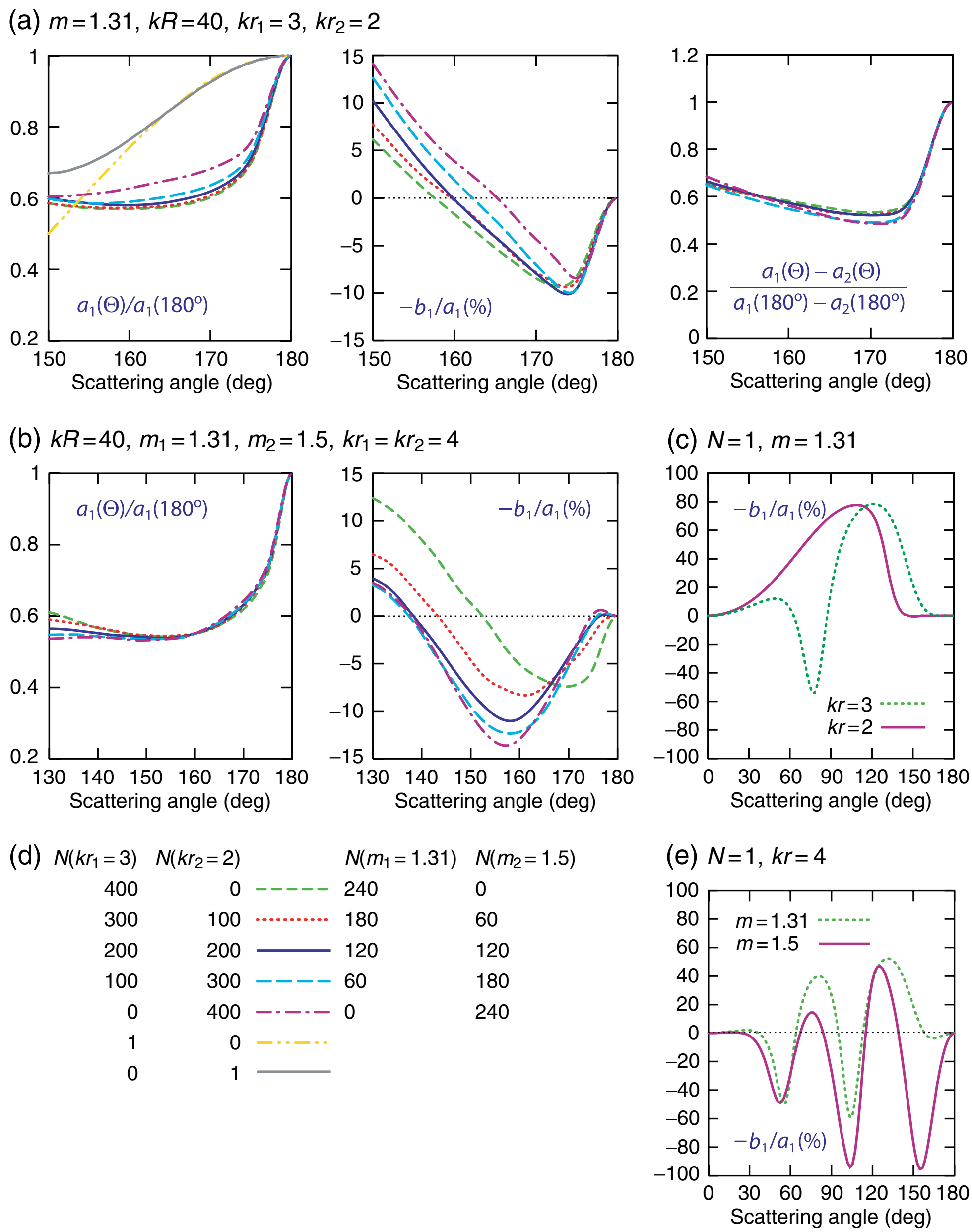

Fig. 2. (Color online) Ratios of the elements of the Stokes scattering matrix. The left- and right-hand parts of the color legend in Fig. 2(d) apply to the plots in Figs. 2(a) and 2(b), respectively.

half-maximum comparable to $1/kR$. The invariance of the phase-function angular profiles and their amplitudes with changing ratio $N(m_1 = 1.31)/N(m_2 = 1.5)$ in the left-hand panel of Fig. 2(b) is especially remarkable. The angular profiles and amplitudes of the cross-polarized intensity peaks in the right-hand panel of Fig. 2(a) are also virtually independent of the ratio $N(kr_1 = 3)/N(kr_2 = 2)$. These traits are clear indications of the fundamental interference nature of the backscattering peaks (see [6,7]).

The middle panel of Fig. 2(a) shows the classical angular profile of the polarization opposition effect (POE) caused by CB [17–19]. The scattering angle at which polarization changes sign from negative to positive decreases with increasing numbers of $kr = 3$ particles. Still, the single-particle polarization of the $kr = 3$ particles [Fig. 2(c)] remains sufficiently Rayleigh-like to make POE work. The horizontal "shelves" of neutral polarization at backscattering angles exhibited by both curves in Fig. 2(c) make the identification of POE in the middle panel of Fig. 2(a) unequivocal (see [19]).

The single-particle polarization curves in Fig. 2(e) are substantially different from those in Fig. 2(c), especially the $m = 1.5$ one. The strong negative polarization observed for the $m = 1.5$ particles at backscattering angles with a deep minimum centered at $\Theta \approx 156°$ obviously dominates the $N(m_2) = 60$, 120, 180, and 240 curves in the middle panel of Fig. 2(b). The $N(m_2) = 0$ curve shows some traits of POE including the angular position of the minimum, but the angular shape of the minimum deviates from those in the middle panel of Fig. 2(b). This is a natural consequence of the strong deviation of the angular profile of the $m = 1.31$ single-particle curve in Fig. 2(e) from the bell-shaped Rayleigh profile [see the curves in Fig. 2(c)].

In summary, the results of our numerically exact STMM computations of electromagnetic scattering by polydisperse discrete random media fully corroborate the universal interference nature of CB. They also illustrate that one manifestation of CB, namely, the POE, is less robust and hence less ubiquitous than the others. Indeed, it has the tendency to weaken and even disappear once the particle size parameter is outside the Rayleigh range, thereby causing single-particle polarization curves to deviate significantly from the classical bell-shaped profile with a strong maximum at side-scattering angles.

While our conclusions are based on computations for "discretely" polydisperse particulate volumes, one can expect them to apply, at least qualitatively, to "continuously" polydisperse discrete random media. We plan to analyze this more challenging model of polydispersity in the future.

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References

1. Yu. N. Barabanenkov, Yu. A. Kravtsov, V. D. Ozrin, and A. I. Saichev, in *Progress in Optics*, E. Wolf, ed. (1991), Vol. 29, pp. 65–197.
2. L. Tsang and J. A. Kong, *Scattering of Electromagnetic Waves: Advanced Topics* (Wiley, 2001).
3. B. A. van Tiggelen and S. E. Skipetrov, eds., *Wave Scattering in Complex Media: From Theory to Applications* (Springer, 2003).
4. M. I. Mishchenko, V. K. Rosenbush, N. N. Kiselev, D. F. Lupishko, V. P. Tishkovets, V. G. Kaydash, I. N. Belskaya, Y. S. Efimov, and N. M. Shakhovskoy, *Polarimetric Remote Sensing of Solar System Objects* (Akademperiodyka, 2010).
5. M. I. Mishchenko, L. D. Travis, and A. A. Lacis, *Multiple Scattering of Light by Particles* (Cambridge U. Press, 2006).
6. M. I. Mishchenko, L. Liu, D. W. Mackowski, B. Cairns, and G. Videen, Opt. Express **15**, 2822 (2007).
7. M. I. Mishchenko, J. M. Dlugach, and L. Liu, Phys. Rev. A **80**, 053824 (2009).
8. Y. Okada and A. A. Kokhanovsky, J. Quant. Spectrosc. Radiat. Transfer **110**, 902 (2009).
9. D. W. Mackowski and M. I. Mishchenko, Phys. Rev. A **83**, 013804 (2011).
10. K. Lumme and A. Panttilä, J. Quant. Spectrosc. Radiat. Transfer **112**, 1658 (2011).
11. J. M. Dlugach, M. I. Mishchenko, L. Liu, and D. W. Mackowski, J. Quant. Spectrosc. Radiat. Transfer **112**, 2068 (2011).
12. D. W. Mackowski and M. I. Mishchenko, J. Opt. Soc. Am. A **13**, 2266 (1996).
13. D. Mackowski, K. Fuller, and M. Mishchenko, ftp://ftp.eng.auburn.edu/pub/dmckwski/scatcodes/index.html.
14. D. W. Mackowski and M. I. Mishchenko, J. Quant. Spectrosc. Radiat. Transfer **112**, 2182 (2011).
15. M. I. Mishchenko, L. D. Travis, and A. A. Lacis, *Scattering, Absorption, and Emission of Light by Small Particles* (Cambridge U. Press, 2002).
16. D. W. Mackowski, J. Quant. Spectrosc. Radiat. Transfer **100**, 237 (2006).
17. M. I. Mishchenko, J.-M. Luck, and T. M. Nieuwenhiuzen, J. Opt. Soc. Am. A **17**, 888 (2000).
18. K. Muinonen, Waves Random Media **14**, 365 (2004).
19. M. I. Mishchenko, J. M. Dlugach, L. Liu, V. K. Rosenbush, N. N. Kiselev, and Yu. G. Shkuratov, Astrophys. J. **705**, L118 (2009).