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Stochastic acceleration of ions driven by Pc1 wave packets

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The stochastic motion of protons and He^+ ions driven by Pc1 wave packets is studied in the context of resonant particle heating. Resonant ion cyclotron heating typically occurs when wave powers exceed 10^{-4} nT^2/Hz. Gyroresonance breaks the first adiabatic invariant and energizes keV ions. Cherenkov resonances with the electrostatic component of wave packets can also accelerate ions. The main effect of this interaction is to accelerate thermal protons to the local Alfven speed. The dependencies of observable quantities on the wave power and plasma parameters are determined, and estimates for the heating extent and rate of particle heating in these wave-particle interactions are shown to be in reasonable agreement with known empirical data.
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I. INTRODUCTION

Pc1, or electromagnetic ion cyclotron (EMIC), waves exhibit frequencies of 0.1 to 5.0 Hz, occur in the plasmasphere, and intensify during magnetic storms. Anisotropic distributions of keV ring current (RC) protons provide a source of free energy for EMIC wave excitation.[1–3] Pc1 events typically last for more than several hours and appear as coherent wave packets with repetitions period of tens of seconds to a few minutes with typical broadband amplitudes reaching values in the range of 1–10 nT. These waves usually exhibit mixed polarization because they belong to two distinct branches: an ion whistler branch, and an Alfven-like branch with a different polarization.[4–7] These wave modes can interact with keV ions, and the intensification of ion fluxes associated with wave events has been shown in *in situ* measurements.[6,8] Along with the expected keV protons, greatly enhanced fluxes of cool protons (30–300) eV have also been observed by Engebretson *et al.*,[4] Pickett *et al.*,[5] and Arnoldy *et al.*[9] Information about He^+ ion energization was obtained from the Geostationary Operational Environmental Satellite (GOES) and has been interpreted as a result of the ion-EMIC wave interaction.[10]

The association of Pc1 wave events with intense ion fluxes merits further attention. Simultaneous increases in measured fluxes and wave fields suggest that the Pc1 waves energize the ions. In this paper, we explain the idea of stochastic particle heating.[11] This idea is based on nonlinear (NL) resonant wave-particle interactions, which can cause stochastic particle motion and consequently particle heating.

The paper is organized as follows. Sec. II considers the polarization and dispersion properties of Pc1 wave packets. Sec. III provides the original equations of motion of particles driven by EMIC waves. Sec. IV touches on the problem of acceleration and energization of ions via gyroresonance with these waves. Sec. V examines proton heating via Cherenkov resonance with the electrostatic component of oblique Pc1 wave packet. Sec. VI discusses and summarizes the main results obtained in this work.

II. DISPERSION OF NONLINEAR Pc1 WAVE PACKETS

The dependence of the Fourier-harmonics of EMIC waves on the wave number k in the range $(kv_A)^2 \ll \Omega_p^2$ is given by the relation[12,13]

$$\omega(k) = kv_A(1 \pm kv_A/2\Omega_p), \tag{1}$$

where the negative sign stands for the Alfven-like branch, Ω_p is the proton gyrofrequency, $v_A = c\Omega_p/\omega_L$ is the Alfven speed, c the light speed, and ω_L the proton Langmuir frequency. Here, $c^2 \gg v_A^2$, and the thermal ion velocity, v_{T_i}, obeys $|\omega - \Omega_i| \gg kv_{T_i}$, and $v_A^2 \gg v_{T_i}^2$, v_{T_i}. EMIC waves are believed to radiate from the ring current region through gyroresonance with keV RC protons. Leaving aside the problems that arise in such models for EMIC wave generation,[14] we take for granted the existence of such waves. The RC origin and dispersion relationship (1) have been confirmed by measuring EMIC wave polarization, velocity, and dispersion.[4,7,15] The contribution of the heavy ion current to wave dispersion is small, and can be neglected provided $c^2 \gg v_A^2$.[4]

As is known, any NL EMIC wave with an NL profile envelope, as well as NL whistlers, can be described by a nonlinear Schrödinger (NLS) equation[16] or its modifications.[17] The NL dispersion relation corresponding the NLS equation takes the form,

$$\omega(k) = \omega(k_0) + \frac{1}{2}\left(\frac{\partial^2\omega}{\partial k^2}\right)_{k_0} \cdot (\Delta k)^2 + \left(\frac{\partial\omega}{\partial a^2}\right)_{k_0} \cdot a^2, \tag{2}$$

where $\omega(k_0)$ is the frequency of the principal mode, Δk the width of the k-spectrum, and a the wave amplitude. One also assumes that the second and third terms in Eq. (2) are small compared to $\omega(k_0)$.[16] The NLS wave equation itself and NL dispersion law (1) remain the same if the relative rate of change of the frequency $\dot{\omega}$, $\dot{\omega}/\omega^2$, and the wave dispersion $\partial k/\partial zk^2$ along the external magnetic field $\mathbf{B}$, are much less than 1. Since $\dot{\omega}/\omega^2$ is of the order of $1/\omega T$ and $\partial k/\partial zk^2 \sim 1/kL$ or $1/kl$,

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where T, L are the time- and spacescales of the wave packet and l is the spacescales of inhomogeneity in B, we write the conditions for the applicability of the NLS equation as

$$1/\omega T, 1/kL, 1/kl = \varepsilon, \ \varepsilon \ll 1. \tag{3}$$

In wave event measurements by remote monitoring the wave spectra, this frequency drift emerges as a consequence of the difference in phase velocities caused by the NL dependence of frequency on the wavenumber k. A more important effect, the so-called self-modulation of the wave, is due to the periodic nature of the wave amplitude, which introduces a time-dependent modulation of the wave frequency. The third term in Eq. (2), proportional to the square of the slowly varying amplitude, describes this effect entirely.[11] For further details, the reader is referred to this paper.[11] Typical values of $\dot{\nu}$ measured by Feygin *et al.*,[15] Mursula,[18] and Pickett *et al.*[5] are $\dot{\nu} \sim (1-3)\times 10^{-2}$ Hz/s, $\nu = \omega/2\pi \sim 1.5$ Hz, so condition (3) is trivially satisfied. So, any NL EMIC wave can be described by a wave solution of an NLS equation conforming to the problem. On the other hand, any wave with an NL envelope has a wide spectrum of strongly bounded Fourier harmonics, and hence can be represented in the form of an NL wave packet. Substantiation of the approach can be found in the monograph,[19] and in the physics of radiation belts (RBs) this approach has been introduced in Refs. 20 and 21. These authors also showed that both representations are identical.[11]

The dispersion relation (1), at least for the Alfven branch of EMIC waves propagating at an arbitrary angle ϑ to the external magnetic field, remains applicable, with $k_z = k\cos\vartheta$ replacing k. If the wave propagates nearly across B, a term $(k_t r)^2$, where r is the ion gyroradius, also emerges in Eq. (1).[13] The subscripts z and t stand for the components of a vector quantity, parallel and perpendicular to the background magnetic field.

Anderson, Erlandson, and Zanetti[22] attributed the left-hand and/or right-hand polarization of these waves to oblique wave propagation. Indeed, the unusual polarization could be caused by the simultaneous generation of the two branches of EMIC wave by RC protons. Another possible explanation for this baroque polarization is related to the transverse-longitudinal nature of the oblique wave in the generic case. Consider low-frequency mixed modes. These perturbations are partially electrostatic and partially electromagnetic, and differ from the usual EM waves in that the former contains a potential electric field E_z^l along B.[13] The relationship between the potential component E^l and electromagnetic E^t can be found using the equation of continuity, two Maxwell equations, and Poisson's equation.[12] From those equations, the following expression for E^l/E^t results:

$$E^l/E^t = (kc/\omega)^2 J^l/J^t,$$

where J is the electric current driven by these low-frequency perturbations. Since $kc/\omega \sim c/v_A$, and $c/v_A \gg 1$, E^l would be non zero even when a strong anisotropy of the electric current, $J^l/J^t \ll 1$, is present. Note that both the low-frequency band of the oblique Alfven mode, the so-called kinetic Alfven wave, and the high-frequency oblique whistler wave have strong potential components that are comparable to the values of the E^t-components.[23–26]

III. ORIGINAL EQUATIONS OF ION MOTION

Consider an ion of charge $q_i = 1$ and mass m_i, which gyrates in a background magnetic field $\mathbf{B}$ and resonantly interacts with a packet of circularly polarized EMIC waves. The Hamiltonian of the problem is of the form

$$H(p_z, z, I, \theta; t) = H_0(p_z, I) + \sqrt{\frac{2\Omega_i I}{m_i}}\frac{2B^w J_1'(k_t r)}{k_z}, \tag{4}$$

$$H_0 = p_z^2/2m_i + \Omega_i I, \tag{5}$$

given on a 4-D manifold called phase space by two pairs of variables, (p_z, z) and (I, θ), where I,θ and p_z are the action, gyro-angle, and p_z-momentum, respectively, B^w is the magnetic component of the wave field, and $J_1'(k_t r)$ is the derivative of the Bessel function with respect to its argument.

We shall assume that the length scales of the inhomogeneities of these fields and the ratio B/B^w are sufficiently great that the standard description of particle motion is applicable. Then, the ion gyration and motion along B are described by

$$\dot{\theta} = \frac{\partial H}{\partial I} = \Omega_i, \quad \dot{z} = \frac{\partial H}{\partial z} = v_z, \quad v_z = p_z/m_i. \tag{6}$$

These equations determine the change of phase of a particle in the wave field,

$$\dot{\psi}(v_z) = k_z v_z + s\Omega_i - \omega, \tag{7}$$

where $s \in Z$, Z is the set of all integers, including null.

In the following, we are concerned only with the dynamics of ions in the vicinity of the principle resonances,

$$\dot{\psi}(v_z) = k_z v_r + s\Omega_i - \omega = 0, \ s = 0; 1. \tag{8}$$

In this case, ψ varies slowly, $\dot{\psi}/\omega\psi \sim \varepsilon$, and therefore the variables p_z and I also vary slowly over one wave period ω^{-1}, $\dot{p}_z/\omega p_z \sim \varepsilon$, $\dot{I}/\omega I \sim \varepsilon$, where ε is a small parameter, $\varepsilon = 1/\omega T$.

Now the equations of motion, $\dot{p}_z = -\partial H/\partial z$, $\dot{I} = -\partial H/\partial\theta$, associated with Hamiltonian (4), become

$$\dot{v}_z = \Omega_i v_t \frac{2bJ_1'(\cdot)}{\sqrt{2\pi\Delta\nu T}}\sin\psi \sum_n \delta(t/T - n), \tag{9}$$

$$\dot{v}_t = s\Omega_i v_A \frac{\Omega_i}{\omega}\frac{2bJ_1'(\cdot)}{\sqrt{2\pi\Delta\nu T}}\sin\psi \sum_n \delta(t/T - n), \tag{10}$$

where all of the perturbation terms, including those describing mirror effects, average to zero except for $s = 1$.[11] We have used here the representation of the NL wave field in the form of the wave packet,

$$B^w(z,\theta,t) = B\frac{b}{\sqrt{2\pi\Delta\nu T}}\cos\psi \sum_n \delta(t/T - n),$$
$$b = B_0^w/B, \quad v_z \le v_A, \ n \in Z, \tag{11}$$

where B_0^w is the peak value of the wave field, $\Delta\nu$ is the width of spectrum, T is the envelope period, and $\delta(\cdot)$ is the Dirac function. We have also applied the relation $\Omega_i I = m_i v_t^2/2$ for the energy of gyromotion, and we assume that the characteristic phase velocity is approximately equal to the Alfven speed.

The phase flow of this system conserves the invariant of motion,

$$v_t^2 - 2s(\Omega_i/\omega)v_A v_z = C, \tag{12}$$

where C is a constant which is determined by the initial resonance condition. The invariant reduces the dimensions of phase space from four to two.

A further simplification of the system is related to the generic behavior of periodic Hamiltonian systems which are usually described in terms of maps. The method of finding these maps involves a transformation g^1 realized by phase flow at one period.[27] By using the equation for particle phase (7), and either equation from sets (9) and (10) along with the invariant of motion (12), we can reduce in general the system to the map g^n,

$$g^n : u_{n+1} = u_n + Q\sin\psi_n,\ \psi_{n+1} = \psi_n + F(u_{n+1}) \quad \mathrm{mod}2\pi, \tag{13}$$

where u is a new action variable, Q the control parameter, differentiable function $F(u)$ describes the shift of phase acquired by the particle, and the values of (u_n, ψ_n) are taken at $t_n = nT$. These transformations constitute the group $g^n = (g^1)^n$ that acts on the 2D-manifold M (phase space) as a dynamical system given by the pair (M, g^n). The problem defined in this way can be treated analytically as well as numerically. First, we observe that the set of NL difference equations (13) is a measure-preserving map for the canonical pair (u, ψ). This can be corroborated by direct computations of the Jacobian $detJ = 1$, $J = \partial(u_{n+1}, \psi_{n+1})/\partial(u_n, \psi_n)$. The qualitative behavior of the system can be understood by calculating the multiplier $|\delta\psi_{n+1}/\delta\psi_n - 1| = QF'$, $F' \equiv \delta F/\delta u$. Provided that $QF' \geq 1$, a local phase instability arises, and the dynamics is thought to become chaotic.[19] A more descriptive portrait of the bifurcation to chaotic behavior is provided by the topology of phase space, which is closely related to the dynamics. The topological modification of phase space is known to occur when the condition $|trJ| - 1 \geq 2$, where trJ is the trace of Jacobi matrix, is fulfilled.[27] Applying this condition to the system gives the criterion

$$QF' \geq 1. \tag{14}$$

If it is valid, the system will exhibit robust chaotic motion on a strange attractor (SA) tightly embedded in phase space. In this case, the phase trajectory explores the entire phase space energetically accessible to it; the upper bound of the SA, u_b, corresponds to the upper value of continuous invariant u-spectrum, and the time of relaxation to this uniform distribution is determined by the equation

$$t_d = 2u_b^2/D, \tag{15}$$

where D is a coefficient of diffusion in u.

IV. STOCHASTIC ION HEATING

Consider the dynamics of ions which interact with Pc1 wave packets via gyroresonance with Doppler-shifted frequency $\omega + k|v_z|$,

$$R = (\Omega_i - \omega)/\omega. \tag{16}$$

Invariant (12) physically associated with the resonance becomes

$$v + w^2/(2\Omega_i/\omega) = R. \tag{17}$$

Both equations are written in the dimensionless variables

$$v = |v_z|/v_A, \quad w = v_t/v_A, \quad R = v_r/v_A. \tag{18}$$

For heavy helium ions, such a physical situation arises when the principal frequency of the wavepacket lies just below the helium gyrofrequency Ω_h.[6,7] In this region, strongly coherent wave signals with repetitive structure are observed. The spectral power of these waves with left-to right handed polarization for small nonzero angles and envelope timeperiods ~ 100 s were measured to be 10–100 nT^2/Hz.

To argue the problem in terms of the map (13), we integrate Eq. (10) and the equation for phase (7) eliminating the variable v_z with the help of the invariant of motion (12), to obtain a set of NL difference equations that map $R \times S$ onto itself,

$$w_{n+1} = w_n + (\Omega_h/\omega)^2(2\pi\nu^2 PT)^{1/2}\sin\psi_n,\ w \in R, \tag{19}$$

$$\psi_{n+1} = \psi_n + (w/2\Omega_h)\omega T w_{n+1}^2 \quad \mathrm{mod}2\pi, \psi \in S, \tag{20}$$

$$P = b^2/\Delta\nu,\ b = B_0^w/B, \tag{21}$$

where P is the spectral power normalized to B^2, and $J_1'(\cdot) = 1/2$ for a quasi-parallel wave.

By using condition (14) and the invariant of motion (12), we find the range of w-values, (w_a, w_b),

$$w_a = \omega/\Omega_h\omega T(2\pi\nu^2 PT)^{1/2},\ w_b^2 = (2\Omega_h/\omega)R, \tag{22}$$

where the dynamics becomes stochastic.

It turns out that the upper bound of the w-spectrum is defined entirely by the dynamical invariant (12), which limits the spectrum. On the other hand, the invariant expressed as a function $w(v)$ describes an anisotropic ion distribution (isodensity level line at a fixed value of R), which exhibits the shape of a horseshoe or pancake in w–v phase space.[28]

The prediction of Eq. (22) agrees reasonably well with the numerically determined w-spectrum from the map in the range of interest shown in Figure 1. Here, we introduce a phase shift $w \to w + w_a$ in the phase-advanced equation to make the resulting set of equations symmetric about the w-axis.

To show that these dynamics are actually chaotic, we define the correlation function of phase by

$$C(i) = (1/N)\sum_{n\in N}\psi(n)\psi(n+i), \tag{23}$$

where i is the step lag and N is the total number of iteration steps. Numerical calculations yield the result shown in

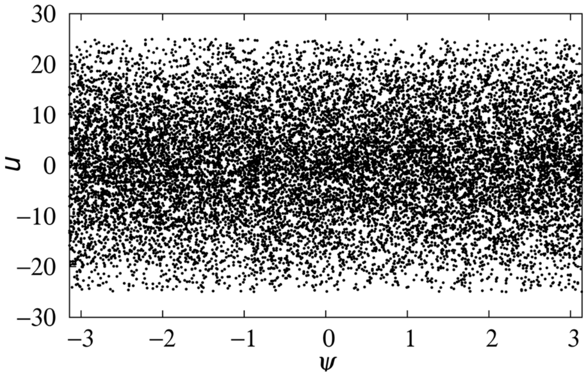

FIG. 1. A stochastic trajectory for systems (19)–(21) at $Q = 5$. The variable w is normalized as $\sqrt{\omega/\Omega_h \omega T} w = u$, $Q = (\omega T \cdot 2\pi\nu^2 PT)^{1/2}(\Omega_h/\omega)^{3/2}$, and in the phase-advanced equation the phase shift $u \to u + u_a$ has been made, $u_a = w_a\sqrt{\omega/\Omega_h \omega T}$, w_a is given by Eq. (22). Initial values of ψ and u are close to zero.

Figure 2, indicating that $C(i)$ falls off rapidly with the number of map iterations.

There is no doubt that this mixing motion takes place on a SA, the lower and upper bounds of which are given by Eq. (22). The stochastic motion leads to the uniform distribution on the SA

$$f(w) = 1/w_b, \tag{24}$$

provided the inequality $w_a \ll w_b$ is valid. Then from Eq. (19), we calculate the coefficient of diffusion in w,

$$D = \pi(\Omega_h/\omega)^4 \nu^2 P. \tag{25}$$

We have employed the definition $D = \langle (u_{n+1} - u_n)^2 \rangle / T$ in deriving (25), where $\langle \cdot \rangle$ denotes the phase average.

Thus, the time of relaxation to the distribution (24), t_d, is found to be

$$t_d = (4R/\pi\nu^2 P)(\omega/\Omega_h)^3, \tag{26}$$

as it appears from the expressions for w_b and D.

In accord with Paulson *et al.*,[7] we take the following parameter values, $v_A/c = 10^{-3}$, $\nu = 0.4$ Hz, $\nu_h = 0.6$ Hz, $T = 100$ s, and $P = 10^{-3}$ 1/Hz, to evaluate observables. Substituting these values into Eqs. (16) and (21) gives $R = 0.5\, w_a = 10^{-2}$, and $w_b = 1.25$, so that the inequalities

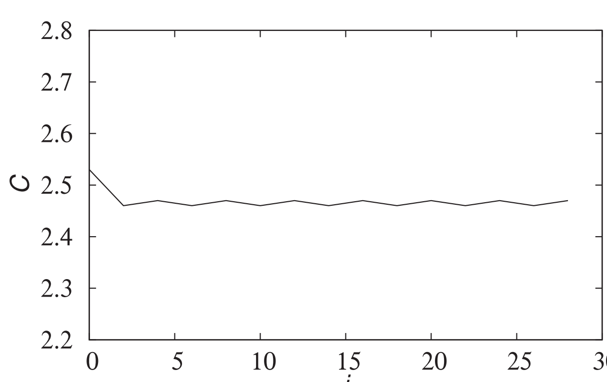

FIG. 2. Correlation function $C(i)$ of systems (19)–(21) with $Q = 5$.

$$w_a \ll R \lesssim w_b \tag{27}$$

are fulfilled. Then in conformity with Eq. (27), we find the energy of He^+ ions in resonance, $E_r = R^2 E_A, E_r = 0.5$ keV, the upper value of energy spectrum, $E_b = w_b^2 E_A (= 3$ keV), which arises out of the invariant, and the relaxation time $t_d = 1 \times 10^3$ s, which result from Eq. (24).

We also consider resonant ion cyclotron heating (RICH) of protons. It is clear that the equations of proton motion are identical in form to the maps (19) and (20), indicating that the system undergoes bifurcation to stochastic behavior at parameter values given by Eq. (22) with Ω_p replacing Ω_h.

Note that condition (27) places some restrictions on the permissible values of P

$$P_\nu \gg P_c,\ P_c = B^2(\omega/\Omega_i)^2/2\pi\nu^2 T(\omega TR)^2, \tag{28}$$

which appear from Eqs. (16) and (22). As a consequence, in sufficiently strong wave fields, the extent of particle heating is determined by the expression,

$$\Delta E/E_A = (E_b - E_a)/E_A = (\Omega_i/\omega)^2 - 1, \tag{29}$$

irrespective of P if $P \gg P_c$.

In the following estimates, we fall back on the paper of Engebretson *et al.*,[4] in which fine patterns of Pc1 wave packet are presented. The authors observed predominantly transverse 1.8 Hz quasiparallel waves with peak amplitudes of 4–6 mV/m measured by the Cluster spacecraft. These intense wave packets with 10 nT^2/Hz spectral power exhibit the shape of repetitive coherent signals with typical envelope timeperiods $T \simeq 25$ s. Applying these values to Eqs. (28) and (29), we find $P_c = 10^{-4}$ nT^2/Hz at $B \simeq 300$ nT, the extent of heating, $\Delta E = 4$ keV at $E_A = 0.5$ keV and $\Omega_p/\omega = 3$. Equation (26) allows us to find the relaxation time $t_d \approx 300$ s, which roughly amounts to 100 timeperiods of wave. Finally, the average heating rate of the process, $\dot{E} = E_b/t_d$, proportional to P and E_A, is about 20 eV/s, as follows from Eqs. (22) and (26). This result agrees well with the experimental data of Arnoldy *et al.*[9] Note that Figure 1 shows that the stochastic motion of particle in waves breaks the first adiabatic invariant μ, where $\mu = mv_t^2/2B$. Consequently, we may only discuss the probable values of μ, and predict by means of $f(w)$ and w_b the typical mean values of μ, $\langle \mu \rangle = (2\Omega_i R/3\omega)(E_A/B)$, $\mu \approx 0.66 \times 10^{-8}$ J/T.

EMIC waves, which are generated by keV RC protons, cause strong localized proton precipitation into the upper atmosphere.[29] The interplay between the resonant part of anisotropic distribution of RC protons and EMIC waves seems to help explain why intense proton fluxes are observed.

Although cyclotron heating of protons is possible only through the resonance, NL coupling with EMIC waves with a principal frequency just above the helium gyrofrequency can effectively accelerate heavy He^+ ions through the resonance

$$R = 1 - \Omega_h/\omega. \tag{30}$$

The equation is written with the preceding notation.

This type of wave-particle interaction obviously conserves the following invariant:

$$v - (\omega/2\Omega_h)w^2 = R. \tag{31}$$

The physical situation can arise, for example, in the interaction of He$^+$ ions with Pc1 wave packets, observed by Engebretson *et al.*[4] and Pickett *et al.*[5]

The dynamical invariants, (17) and (31), are qualitatively different, and as a consequence, the particle behaviors can be substantially different. However, when $v_z < v_A$, the local topologies of these invariants are identical. Direct analysis of the set (24) in this range shows that the equations of particle motion would look exactly like the map (19) and (20), exhibiting behavior with some features similar to that of the systems (19) and (20).Unlike the preceding case, the dynamical invariant (31) no longer limits the v-spectrum, consequently, we can no longer use the representation (11) in the range where the particle velocity along the direction of external magnetic field exceeds the Alfven speed substantially. Instead, we should use the representation

$$B^w = (B_0/\sqrt{\Delta kL}) \sin\psi \sum_n \delta(z/L - n), \tag{32}$$

where L is the spacescale of wave packet and Δk is the width of packet in k-space.[11,19]

One then assumes that $\Delta kL = 2\pi\Delta\nu T$, where $\Delta\nu$ is the width of frequency spectrum, $L = v_{AT}$, and approximates the invariant (31) by the equation,

$$w^2 = (2\Omega_h/\omega)v. \tag{33}$$

Approximations of this kind are inevitable when dealing with complex dynamical systems.

Substituting Eq. (32) in Eq. (9) gives the following equation:

$$dv_z = \Omega_h v_t \sqrt{\frac{P}{2\pi T}} \sin\psi \frac{L}{v_z} \sum_n \delta(z - nL)dz, \tag{34}$$

where we have used the equation $dt = dz/v_z$, dv_z and dz are considered to be 1-forms, and L/v_z is the time taken by the particle to pass through the wave packet.

One eliminates v_t from the equation with the help of Eq. (33), and integrates Eq. (34) along with the equation for particle phase, to obtain the map

$$u_{n+1} = u_n + (3Q/2)\sin\psi_n,\ \psi_{n+1} = \psi_n + \omega TRu_{n+1}^{-2/3} \mathrm{mod} 2\pi, \tag{35}$$

given by the canonical pair (u, ψ), where

$$u = v^{3/2},\ Q = (\Omega_h/\omega)^{3/2}(4\pi\nu^2 PT)^{1/2}. \tag{36}$$

Now the condition of topological modification applied to Eq. (35) determines the upper bound, u_b, of the u-spectrum

$$u_b = (\omega TRQ)^{3/5}. \tag{37}$$

We will take, according to Engebretson *et al.*,[4] $\Omega_h/\omega = 3/4, \nu = 1.8\,\text{Hz}$, $P_\nu = 10\,\text{nT}^2/\text{Hz}$, $B = 300\,\text{nT}$, and $T = 25\,\text{s}$, to obtain $R = 0.25$, $Q \approx 0.3$, and $v_b \approx 4$. Note that the condition $v_b \gg R$ is easily satisfied.

The result (37) is borne out numerically, and Figure 3 shows an almost uniform u-spectrum. Consequently, the relaxation time to the local equilibrium $f(u) = 1/2u_b$ is $t_d = 2u_b^2/D$, where the coefficient diffusion D in u is determined by

$$D = Q^2/2\tau. \tag{38}$$

We take into account the fact that the timeperiod $\tau = L/v_z = T/v$, over which the phase averaging is performed, hinges on the velocity v_z, to find

$$D = D_0 u^{2/3},\ D_0 = (9/2)(\Omega_h/\omega)^3\pi\nu^2 P,\ t_d = 2v_b^2/D_0. \tag{39}$$

By making estimates of the extent and rate of heating due to this type of interaction, it has been found that $t_d \approx 1 \times 10^4$ s, $E_R = R^2 E_A = 0.5$ keV, $E_b = (v_b^2 + w_b^2)E_A = 40$ keV, and $\dot{E} = E_b/t_d \sim 4$ eV/s at $E_A = 2$ keV.

The values of E_R and E_b define the boundaries of the energy spectrum, which, in addition to their dependence on E_A, depend on the wave power as

$$t_d \sim P^{-0.6},\ E_b \sim P^{0.4},\ \dot{E} \sim P. \tag{40}$$

We have shown above that it is impossible to accelerate protons out of the thermal plasma via gyroresonances, one needs to invoke heating from oblique Pc1 waves via a Cherenkov

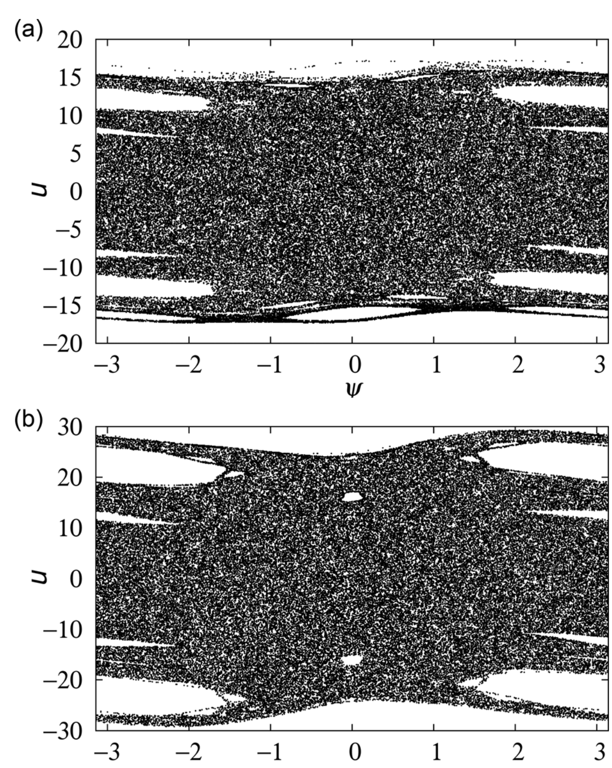

FIG. 3. A stochastic trajectory for system (35) at (a) $Q = 1$, (b) $Q = 2$, $\omega T = 100$.

resonance with the potential component of an Alfven-like wave.

V. PROTON HEATING VIA CHERENKOV RESONANCE

The injection problem of heating sufficiently energetic protons remains unsolved. Pc1 wave packets oblique to **B** have been observed frequently.[4,5] Further comments on this possibility were provided in Section II, and here we discuss its consequences.

The Hamiltonian of the problem is

$$H(p_z, z, t) = p_z^2/2m_i + \Omega_i I + U(z,t)J_0(k_t r), \tag{41}$$

where $J_0(\cdot)$ is the Bessel function and the function $U(z,\ t)$ describes the spatio-temporal structure of potential wave field

$$U(z,t) = (U_0/\sqrt{2\pi\Delta\nu T})\cos\psi \sum_n \delta(t/T - n),$$
$$\psi = k_z z - \omega t,\ v_z \le v_A, \tag{42}$$

and U_0 the peak value of U.

It is apparent that $I = \text{const}$ is the integral of motion which, for simplicity, we set equal to zero and therefore, $J_0(\cdot) = 1$.

The equations of proton motion, related to the Hamiltonian, are

$$\dot{v}_z = k_z(U_0/m\sqrt{2\pi\Delta\nu T})\sin\psi \sum \delta_n, \tag{43}$$

$$\dot{\psi}(v_z) = k_z v_z - \omega, \tag{44}$$

and the condition of the Cherenkov resonance arises from Eq. (44)

$$R - 1 = 0, \tag{45}$$

written with the preceding notation.

In the Cherenkov resonance, slow protons, $v_z \le v_A$, gain energy by picking up wave energy in multiple encounters with Pc1 wave packets. On the other hand, the mechanism of Landau wave damping limits the energy spectrum from below, so the lower boundary of spectrum is typically defined by the condition, $E \gg T_i, T_i \sim 1$ eV.

Then, as above, we reduce the equations of motion (43) and (44) to the map

$$v_{n+1} = v_n + \omega T(U_0/m_p v_A^2\sqrt{2\pi\Delta\nu T})\sin\psi_n,\ \psi_{n+1}$$
$$= \psi_n + \omega T v_{n+1} \quad \text{mod} 2\pi, \tag{46}$$

where $v = v_z/v_A$. Then, we recognize in the system (44) the familiar standard map[30] that describes the transition of the system to global chaotic behavior provided the magnitude of wave field exceeds the value, U_c,

$$U_c = m_p v_A^2\sqrt{2\pi\Delta\nu T}/(\omega T)^2, \tag{47}$$

and consequently, $E_c^l = k_z U_c$. In deriving Eq. (47), we have again used condition (14). Now we estimate E_c^l for typical parameters,[4] $E_c^l \approx 1\times10^{-3}$ mV/m. It is known that the potential waves are associated with fluctuations of the charge density δn,[12] and that the magnitude of the wave field and the fluctuation level $\delta n/n$ are related by the Poisson equation $U_0/mc^2 = (\omega_p/\omega)^2(v_A/c)^2(\delta n/n)$, where ω_p is the ion plasma frequency. Equating this expression to Eq. (47) yields $\delta n/n = (\omega/\omega_p)^2(\sqrt{2\pi\Delta\nu T}/(\omega T)^2)$, $\delta n/n \sim 10^{-9}$, typically. Thus, we have learned that typical values for the potential Pc1 waves seem to be relatively small. Reliable information about its values is still absent.[4]

Calculating the rate of diffusion at $U \ge U_c$ yields

$$D = (U/U_c)^2/2(\omega T)^2 T, \tag{48}$$

so that the particle heating rate is given by the expression $\dot{E} = E_A D$ and $\dot{E} \sim 1$ eV/s at $E_A = 0.5$ keV, $U/U_c = 100$, and $T = 25$ s.

Note that the heating rate in this energy range depends on the wave magnitude as $\dot{E} \approx (U/U_c)^2$ and the process of heating in Cherenkov resonances is sufficiently effective because the heating time is roughly several tens of wave packet timeperiods.

Now, we proceed to the problem of the dynamics of fast protons driven by the potential Pc1 wave packet. In this case, the resonant wave-particle interaction follows in the interchange of energy between waves and particles.[12] Thus, the mechanism serves as a limit to the particle heating that can be achieved. In order to derive equations describing the dynamics of fast protons, whose particle speed is larger than the Alfven velocity, $v_z > v_A$, we have to represent the potential field of the wave packet in the form (32), where U_0 replaces B_0^w. In this way, we formalize the problem in terms of the map,

$$u_{n+1} = u_n + Q\sin\psi_n, \quad \psi_{n+1} = \psi_n + \omega T/\sqrt{|u_{n+1}|}\text{mod}2\pi, \tag{49}$$

where the new variable u and the control parameter Q are defined by

$$u = v^2,\ v = v_z/v_A,\ Q = 2(U/U_c)/\omega T. \tag{50}$$

By the usual methods, we find the upper bound of the u-spectrum,

$$u_b = (U/U_c)^{2/3}, \tag{51}$$

so that the upper boundary of energy spectrum is given by $E_b = u_b E_A(\sim 10\text{ keV})$ at $U/U_c = 100$, and $E_A = 0.5$ keV. If we choose $v_A/c = 4\times10^{-3}$, the value agrees rather well with Ref. 4 and E_b could be equal to $\sim$160 keV.

Numerically integrating the initial condition of the map over 10^5 map iterations, we obtain $u_b = (U/U_c)^{2/3}$ (Figure 4), indicating, as would be expected, the dominance of chaotic motion on the strange attractor. Then, one calculates the correlation function $C(i)$, given by Eq. (23) with the average taken over 10^5 steps. Figure 5 shows that the function $C(i)$ exhibits a very strong chaotic property of the motion, i.e., a complete decorrelation in a few map periods. Thus, the motion has been shown to be mixing, with the

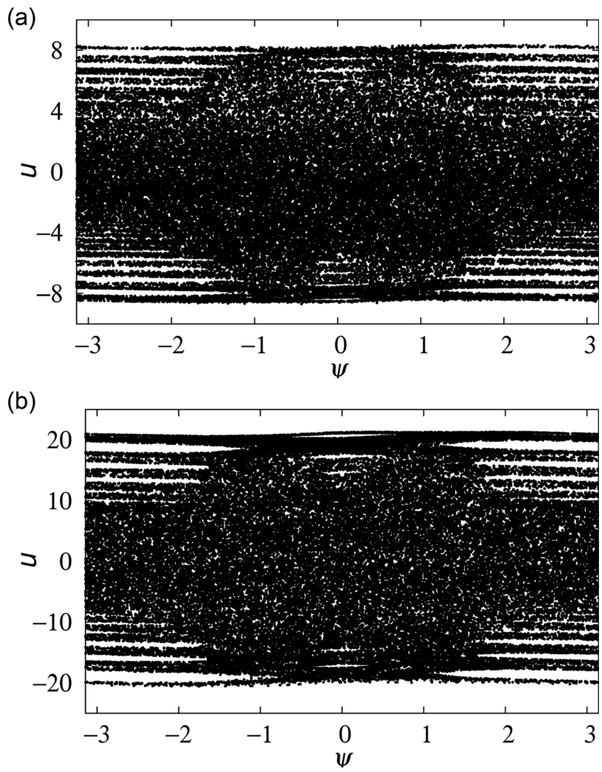

FIG. 4. Phase space $u-\psi$ for map (49) with $\omega T = 300$ at (a) $Q = 27$, (b) $Q = 100$.

relaxation time to the invariant distribution $f(u) = 1/2u_b$ given by $t_d = 2u_b^2/D$, $D = Q^2/2\tau$, where D is the coefficient of diffusion, and $\tau = T/v$ is the period of averaging as a function of the particle speed. With respect to the definitions (50) and (51), we find

$$t_d = T(\omega T)^2(U_c/U). \tag{52}$$

As appears from Eqs. (51) and (52), the time of diffusion and the average heating rate $\dot{E} = E_b/t_d$ are about 100 T and $E_A/10T$ at $U/U_c \sim \omega T \sim 100$, respectively. Note that these results show the following dependencies of the observable quantities on the wave power:

$$E_b \propto (U/U_c)^{2/3}, \quad t_d \propto (U_c/U), \quad \dot{E} \propto (U/U_c)^{5/3}. \tag{53}$$

So, we conclude that the mechanism for heating of cool protons via Cherenkov resonance with the potential component of Pc1 wave packet appears to be much more likely than heating by any other, although the question remains open.

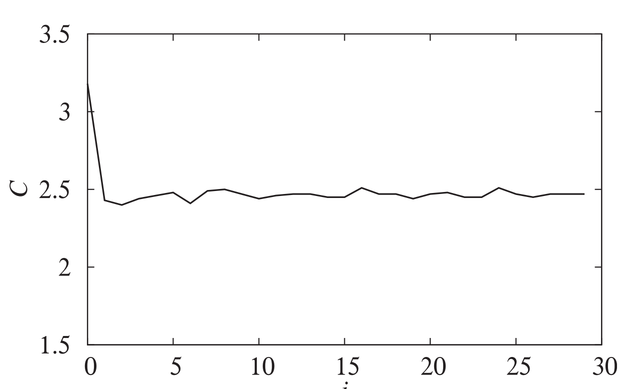

FIG. 5. Correlation function $C(i)$ of system (49) with $Q = 27$.

VI. CONCLUSION

A mechanism for stochastic ion acceleration based on the resonance interaction of ions with nonlinear EMIC waves has been proposed. The dynamics allow stochastic diffusion in phase space to occur, and resonant particles are energized in multiple collisions with EMIC wave. In ion cyclotron heating, the heating is accomplished by resonances between the gyrofrequency and Doppler-shifted wave frequency. In sufficiently intense Pc1 wave packets, the spectral power of which typically exceeds 10^{-4} nT^2/Hz, the stochastic diffusion leads to a new local equilibrium with a wide compactly supported spectra of observables, the signature of chaotic motion. We have shown numerically and analytically the existence of a chaotic solution as a consequence of the resonance wave-particle interaction.

Another area of considerable interest is pitch angle scattering and precipitation of RB ions into the upper atmosphere. A detailed treatment of the problem will be the topic of future investigations. Another mechanism that has been studied to explain ion heating is the Cherenkov resonance with the electrostatic component of an oblique EMIC wave. We estimate the critical magnitude of the potential wave field E_c^l to be $E_c^l \sim 10^{-3}$ mV/m typically, and find it to be sufficient to account for the effect of proton heating via Cherenkov resonance. However, it is not known whether these potential EMIC wave packets have sufficient intensity. It is highly desirable to determine whether the wave magnitudes can be significantly larger than E_c^l. The main effect of the wave-particle interaction is to accelerate the bulk plasma protons to the local Alfven speed. Typical Alfven speeds are high enough, and E_c^l is small enough to be of interest for RB proton acceleration.

The dynamical laws governing the time evolution of a particle in wave-particle interactions lead to a set of nonlinear difference equations. This set, along with an invariant of motion, describes the appearance of strange attractors in a certain parameter range. In this way, the dynamical system can exhibit persistent chaotic behavior. The intensional theoretic structure of the model allows it to both interpret experimental data and predict the behavior of the system in different physical situations. The model has been used to estimate typical values of observables which are consistent with empirical data.

We conclude that these combined mechanisms of resonant wave-particle interaction are most probably responsible for the appearance of RB ions in the energy range from tens of eV to hundreds of keV.

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