Evolution of the spectrum of solar wind velocity fluctuations from 0.3 to 5 AU

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[1] Recent work has shown that at 1 AU from the Sun, the power spectrum of the solar wind magnetic field has the −5/3 spectral slope expected for Kolmogorov turbulence but that the velocity has closer to a −3/2 spectrum. This paper traces the changes in solar wind velocity spectra from 0.3 to 5 AU using data from the Helios and Ulysses spacecraft to show that this is a transient stage in solar wind evolution. The spectrum of the velocity is found to be flatter than that of the magnetic field for the higher frequencies examined for all cases until the slopes become equal (at −5/3) well past 1 AU when the wind is relatively non-Alfvénic. In some respects, in particular in the evolution of the frequency at which the spectrum changes from flatter at larger scales to a "turbulent" spectrum at smaller scales, the velocity field evolves more rapidly than the magnetic, and this is associated with the dominance of the magnetic energy over the kinetic at "inertial range" scales. The speed of the flow is argued to be largely unrelated to the spectral slopes, consistent with previous work, whereas high Alfvénicity appears to slow the spectral evolution, as expected from theory. This study shows that, for the solar wind, the idea of a simple "inertial range" with uniform spectral properties is not realistic, and new phenomenologies will be needed to capture the true situation. It is also noted that a flattening of the velocity spectrum often occurs at small scales.

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1. Introduction

[2] The slope of the spectrum of interplanetary fluctuations is important in that it gives us evidence for the dynamics of their evolution. Much prior work has supported the idea that the Kolmogoroff phenomenology for isotropic fluid turbulence [*Kolmogorov*, 1941] provides the correct prediction for the spectrum of fluctuations in the "inertial range" of scales in which both large-scale stirring and small-scale dissipation are unimportant. While many power spectra had been calculated before, *Matthaeus and Goldstein* [1982] carefully chose three stationary, uniform intervals of data from different radial distances of the Voyager spacecraft from the Sun to calculate the power spectrum using two methods, with the result that the magnetic and total energy spectra had a spectral indices (the slope, α, in a log power versus log frequency plot with $P \propto f^{\alpha}$) very near to the Kolmogoroff value of −5/3 with error estimates (~0.1) that clearly ruled out value of −3/2 predicted for the case of an MHD fluid embedded in a strong magnetic field [*Kraichnan*, 1965]. However, they pointed out that others had found other slopes for solar wind spectra, and that the assumptions of the Kolmogoroff theory, which was for an isotropic fluid, were not valid for the solar wind case. They also noted the strong tendency for the magnetic power in the inertial range to be larger by about a factor of two than the power in the velocity, a fact that had been noted in the earliest papers on Alfvén waves in the solar wind [e.g., *Belcher and Davis*, 1971].

[3] Other careful determinations of solar wind spectra used the magnetic field [*Bavassano et al.*, 1982; *Horbury and Balogh*, 2001] or, increasingly, the "Elsässer variables" $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{b}$ where *xt* $\mathbf{b}f\,\mathbf{b} = \mathbf{B}/\sqrt{(4\pi\rho)}$ and $\mathbf{v}$ is the velocity, $\mathbf{B}$ the magnetic field, ρ the density, and $\mathbf{b}$ is the magnetic field in units of the Alfvén speed [e.g., *Marsch and Tu*, 1990a, 1990b; *Grappin et al.*, 1990]. The conclusion was clear: the magnetic power spectrum is relatively flat, with a spectral index often near −1 at lower frequencies, and the spectral slope steepens to the Kolmogoroff value at higher frequencies. (The −1 slope is not important here, only that the slope is flatter at low frequency [cf. *Nicol et al.*, 2009].) The roll-over (sometimes a sharper break) between the two spectra, which is near 10^{-2} Hz in the most Alfvénic wind at 0.3 AU, moves to lower frequencies with increasing radial distance, indicating a turbulent evolution involving increasingly larger scales. It was also noted in these works that the slow wind frequently was more "highly evolved" in that the −5/3 spectrum already extended to lower frequencies in regions closer to the Sun where the high-speed wind had an extended region of shallow slope. The parcels of wind that showed the more rapid evolution were also less Alfvénic, and this is consistent with studies [*Verma et al.*, 1996, and references therein] that

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demonstrated that the Alfvénic correlation ("cross-helicity"; see below) was a dominant factor in determining the cascade rate of the turbulence. Highly Alfvénic slow-speed regions do exist, and these do not show the rapid spectral evolution [*Roberts*, 2010]. The above observational studies [see also *Roberts*, 1989] determined that the amplitude evolution below the break point was consistent with a dissipationless ("WKB") evolution of the fluctuations, so the interpretation was that while the Kolmogoroff range of the spectrum looks like a turbulent inertial range in which the nonlinear terms are much larger than the dissipation, it is not true that the spectrum is a steady state cascade from a stirring at large scales, but rather a decaying turbulence that proceeds by using up the large-scale energy, dissipating it at small scales, and thus decreasing faster than the WKB prediction. (The term "inertial range" will be used here to refer to the region in which the slope of the spectrum has steepened to something in the range of conventional expectations for a turbulent cascade, but this paper does not presume to demonstrate that this range meets the criteria of any more physically motivated definition of that range.)

[4] More recently, *Podesta et al.* [2006, 2007] studied the magnetic field and velocity spectra separately, using the high-resolution data available from the 3DP instrument on the Wind spacecraft. They concluded that the most common case was that the magnetic spectrum had the expected −5/3 slope, but that the velocity had a shallower slope, very nearly equal to the value of −3/2 expected for a situation with a dominant mean magnetic field. Here we examine the possibility that this surprising result does not represent the asymptotic state of the spectrum, but that the shallower slope represents a transient state in a turbulent system that has not yet reached steady state. The work below provides direct evidence that this is in fact the case. The dominance of the magnetic spectra at high frequencies masks the differences between the magnetic and velocity spectra in previous studies where the two variables were combined. The present work also shows that the velocity, while slower in its spectral steepening, is faster in the evolution of its inertial range to larger scales, and this provides another way to look at the magnetic dominance at small scales. This work will take the magnetic spectral evolution as well-known, but will display magnetic spectra for comparison to the velocity spectra. All of the present magnetic spectra are consistent with previous results. This paper is concerned with the empirical evolution of the spectrum as a function of cross-helicity and heliocentric distance, and does not presume to provide a theoretical understanding.

2. Data Sets and Analysis Procedure

[5] The analysis here extends that of *Podesta et al.* [2006, 2007] by using data from 0.3 to 5 AU to trace the evolution of the spectra. In the inner heliosphere, Helios data made available by R. Schwenn, which include plasma data from his instrument and magnetic field data from F. Neubauer's instrument, provide a combined data set at 40.5 s resolution. These data are available at http://nssdcftp.gsfc.nasa.gov/spacecraft_data/helios/helios2/merged/he2_40sec/ or http://vho.nasa.gov. For the region outside 1 AU, the analysis will show that the combined "COHOWeb" hourly-averaged Ulysses data set (http://cohoweb.gsfc.nasa.gov/form/ulysses.html and ftp://nssdcftp.gsfc.nasa.gov/spacecraft_data/ulysses/merged/), which includes plasma data from D. McComas and magnetic field data from A. Balogh, provides sufficient resolution for the larger scales involved farther from the Sun. This combined data set, which can easily be read into computer memory all at once, also provides a convenient means for doing a statistical study of the spectral slopes. Much higher-resolution Voyager data have yielded similar results for the outer heliosphere, but due to the substantial data gaps in the latter data sets, this work will present only the Ulysses results.

[6] Studies of solar wind evolution typically suffer from poor sampling, in that we do not frequently see the same conditions multiple times at a given radial distance. For the inner heliosphere, this study focuses on the fortunate circumstance, exploited by many others, that the primary mission of Helios 2 sampled a set of very regular streams that recurred four times, at the solar rotation cadence, thus providing a view of very similar plasma at four different radial distances. (Two of these are used below.) These streams are typical of what would be expected based on simple extrapolations of 1 AU data in terms of density, speed and temperature. At each of the distances studied, for both Helios and Ulysses, the most extreme cases of low and high Alfvénicity were chosen that could be found consistent with having a long, uniform data interval. The time resolution of the data was chosen to facilitate the study of many intervals at high enough cadence to see the evolution of the spectrum including some rough measure of the low frequencies and a well-resolved spectrum in the range after the low-frequency "break" (usually a smooth roll-over) to a higher spectral index. For the Helios case, this required the highest-resolution plasma data available; even higher would be desirable, but the main points can be made without this. This work shows that, for the velocity field in the outer heliosphere, hour-averaged data are sufficient to capture this break in index, with a full decade in the steeper slope range. The latter is not true for the magnetic field, but previous work (see citations above) provides the result that the −5/3 slope region observed here over a small range in frequency extends to higher frequencies. There are higher resolution velocity data (typically 4 or 8 min) available for Ulysses, and these will be used in one instance below and in future, more detailed, studies. The frequency range shown here for Ulysses overlaps by one decade with the spectra shown by Podesta et al.; in the latter case, the spectral slope changes very little from this range to higher frequencies.

[7] This analysis is based on conventional Fast Fourier Transforms (FFTs) of subsets of the above data sets. In each case, the power spectrum of all the components are summed to give the total power in the magnetic or velocity spectrum (the "trace of the power spectral matrix"). The spectra below are smoothed logarithmically by averaging values in a bin of ±20% in frequency range around every point in frequency. The spectra all involve 2000 to 3000 points, and thus at low frequencies, where no smoothing occurs, the statistical errors are large, but at higher frequencies up to hundreds of points are averaged leading to very small errors. Generally, the 1-sigma error bars are given by the deviations of the curves from a straight line [see *Podesta et al.*, 2007], and thus are very small at the smallest scales examined here. The largest scales are kept, despite their poor statistics, because they make it easier to see the changes in spectral slope that

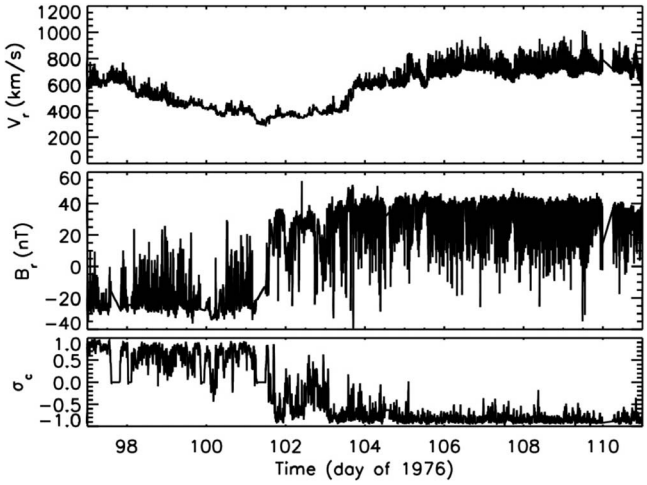

Figure 1. The solar wind speed, radial magnetic field, and velocity-field correlation ("normalized cross-helicity" or "Alfvénicity") at the 15 min scale for 11 days of Helios 2 data in 1976. Days 104–110 are highly Alfvénic wind, whereas day 102 is an example of a relatively non-Alfvénic region. The resolution is 40.5 s.

are central here; they are representative of what is observed when longer intervals are used, but longer intervals are usually not as uniform, so they were not used. In all cases we will use Alfvén speed units for the magnetic field, according to the formula $\mathbf{b} = 21.8\mathbf{B}/\sqrt{(1.2n)}$ with $\mathbf{b}$ in km/s, $\mathbf{B}$ in nT, n (the proton number density at each point) in cm^{-3} and with the factor of 1.2 providing a nominal inclusion of 5% Helium by number. (The latter factor makes no significant difference in the results.) This allows both magnetic and velocity spectra to be plotted with the same units, and allows direct comparison of the relative power levels.

[8] The "normalized cross-helicity" [see *Matthaeus and Goldstein*, 1982], which measures the correlation between the velocity and magnetic fluctuations, is known to have a strong influence on the evolution of MHD turbulence (see, e.g., *M. L. Goldstein et al.* [1995] and *Bruno and Carbone* [2005] for the general context of the work presented here, and *Verma et al.* [1996] for a detailed discussion]. Thus, we use the correlation measure $\sigma_c = 2 \langle \delta\mathbf{v} \cdot \delta\mathbf{b} \rangle / (\langle \delta\mathbf{v}^2 \rangle + \langle \delta\mathbf{b}^2 \rangle)$, with the angle brackets denoting an average at a chosen scale, to organize the discussion. Specifically, the variations were determined by subtracting a running mean over a given timescale, and then the indicated averages were performed over the same timescale. This quantity is also known as the "Alfvénicity" since it is equal to ±1 for pure Alfvén waves propagating along (−) or opposite (+) to the direction of the background magnetic field. Highly correlated flows in this sense evolve more slowly in MHD turbulence, with the extreme case being no nonlinear interactions and thus no turbulent cascade for purely Alfvénic fluctuations [see *Verma et al.*, 1996, and references therein], and this effect will be shown to be consistent with the results below.

3. Example Spectra From 0.3 to 5 AU

[9] Figure 1 shows speed, radial magnetic field, and Alfvénicity from a well-studied interval of Helios 2 data. The high-speed stream here is one of the most Alfvénic intervals ever measured in the solar wind and is the best example we have of a simple, corotating stream as close to the Sun as we have measured (0.3 AU). The spectrum of the highly Alfvénic ($\sigma_c \approx 0.9$) interval on day 106 of 1976 is shown in Figure 2, along with a line for a reference f^{-1} spectrum that is included only to aid the eye in determining the flatness of the slope, not as a theoretical prediction or empirical fit. The lack of a roll-over in the magnetic spectrum to high frequencies was noted previously [*Bavassano et al.*, 1982] and the approximate equality of the magnetic and kinetic energies was noted by *Marsch and Tu* [1990a, 1990b]. A small part of the inertial range may be visible at the highest frequencies for the magnetic field in this plot, although Bavassano et al., found that it clearly exhibits a −5/3 spectrum above 10^{-2} Hz. Unfortunately, there is no higher-resolution data for the velocity. The most important point in this context is that the velocity spectrum is flatter than the magnetic at small scales, and does not roll-over to a −5/3 spectrum. Its spectral exponent is −1 or flatter, apart from a likely random dip at around 10^{-3} Hz. This is the only case in this study where the power in the velocity is larger than that in the magnetic field at the smallest scales. At larger scales, the interval is highly Alfvénic with equal kinetic and magnetic energies to within likely uncertainties. This interval provides us as close as we have to the pristine spectra from the Sun. Other intervals near this one show some differences, with, for example, a yet more nearly −1 low-frequency spectrum at low frequencies and some steepening of the velocity spectrum for day 107, but the qualitative features remain the same. Note that *Roberts et al.* [1987] give an example of highly Alfvénic low-speed wind at 0.3 AU, so high speed is not uniquely associated with high cross-helicity. The Alfvénic low-speed wind interval studied there has power spectra (not shown here, but see *Roberts* [2010]) very similar to Figure 2, with quite flat, equal velocity and magnetic spectra up to the highest frequency range, where the velocity becomes dominant, again with no significant roll-over. This implies that the spectra in Figure 2 are not unusual

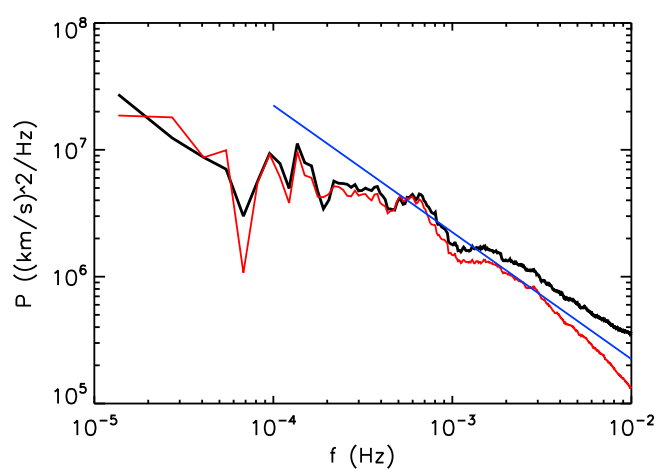

Figure 2. The power spectra of the velocity (black and thicker line) and magnetic field (red, thinner) for the Alfvénic region ($\sigma_c \approx 0.9$) in Helios data from day 106 of 1976, near 0.3 AU. The blue solid line has a spectral index of −1.

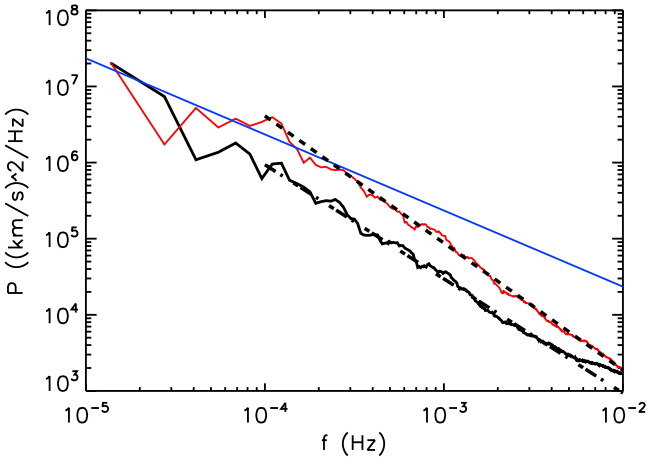

Figure 3. The power spectra of the velocity (black, thicker) and magnetic field (red) for the low-speed, relatively non-Alfvénic region (σ_c near 0.2) in Helios data from day 102 of 1976, near 0.3 AU. Reference spectra have indices of −1 (dark blue, solid), −5/3 (dashed), and −3/2 (dot-dashed).

for 0.3 AU, and that the speed of the wind is not directly associated with the spectral index, as will be shown for other cases below [see also *Tessein et al.*, 2009, Figure 3].

[10] The question naturally arises whether the dominance of the velocity power at small scales is somehow an artifact of measurement or analysis. This is ruled out by various considerations. First, the amplitudes of the velocity fluctuations in this interval are tens of km/s, easily measured by the instrument, and much larger than what appear to be noise-free spectra of lower amplitude in other cases. There is certainly no digitization noise. The spectrum is not characteristic of the aliasing of power from higher frequencies. If the magnetic power continued, even at a flatter spectrum, beyond the range shown, the folding of the unmeasured high-frequency power back into the measured range would give a factor of two increase at the Nyquist (highest) frequency, and much smaller corrections below that. The measured differences between the two spectra are much larger than this. There is also no reason to believe that the velocity spectra would be strongly aliased, as the data reduction involves averaging that will make the instrument insensitive to higher frequencies. Thus, the differences between the velocity and magnetic spectra at high frequencies seen here are almost certainly real. The flattening of the velocity spectrum compared both to itself at lower frequencies and to the magnetic field at comparable frequencies will be seen in a number of cases below, and while those case are sometimes less clear-cut, the above and other reasons support the reality of these differences. Note that such spectral flattenings have been found in the "inward" Elsässer spectra [*Marsch and Tu*, 1990a], in density spectra measured in situ [*Marsch and Tu*, 1990b] and close to the Sun in radio scintillations [*Coles et al.*, 1991], and in the spectrum of the magnitude of the interplanetary magnetic field [*Bavassano et al.*, 1982]. It seems probable that such flattenings are related, perhaps through compressive effects, but this is not known. In the present context, the point is simply to characterize the differences between the velocity and magnetic spectra. The spectrum of the magnetic field components does not show the flattening seen frequently here in the velocity.

[11] In contrast to the case in Figure 1, the slow wind ahead of the stream interface, on day 102, provides an example of a relatively non-Alfvénic interval at the same distance. Figure 3 shows the magnetic and velocity spectra for this case, now along with reference lines representing spectra with spectral slopes of −1, −3/2 and −5/3. The magnetic spectrum is dominant in this case, as found to be typical farther out. Note that the velocity has a flatter spectrum here as well, but now it is close to the −3/2 spectrum reported by Podesta et al. at 1 AU, whereas the magnetic field has the −5/3 index expected from previous work. Note that the velocity spectrum flattens and eventually intersects the magnetic spectrum, but the turnover (if any) to a flatter spectrum at low frequencies is arguably at a larger scale than for the magnetic spectrum (at 10^{-4} Hz); clearer cases will be seen below.

[12] Moving out to near 1 AU, using instances of the wind in the same corotating regions seen three rotations earlier, we again see examples of Alfvénic high-speed wind and non-Alfvénic low-speed wind. The spectrum from the Alfvénic wind, which has an average speed of 650 km/s, is shown in Figure 4 along with the same reference spectral lines. We see a roll-over in the magnetic spectrum to a −5/3 slope at a lower frequency than in Figure 2 (around 10^{-3} Hz), and the velocity spectrum has fallen below the magnetic, except at the smallest scales. The velocity spectrum still is not as "evolved" as that in Figure 3, in that it is flatter than −3/2 and closer to equipartition with the magnetic spectrum; farther out and for common 1 AU conditions, a factor of two or more between the two spectra is typical. (This is discussed as the "Alfvén ratio" problem in the study by *Matthaeus and Goldstein* [1982]; see the introduction above.) As discussed in the context of Figure 2, the velocity spectrum in Figure 4 (see also Figure 5) flattens at higher frequencies; this may be significant, but here we note only that it is not due to aliasing, and that it is quite different behavior from the magnetic spectrum, which does not show such a flattening. (The case in

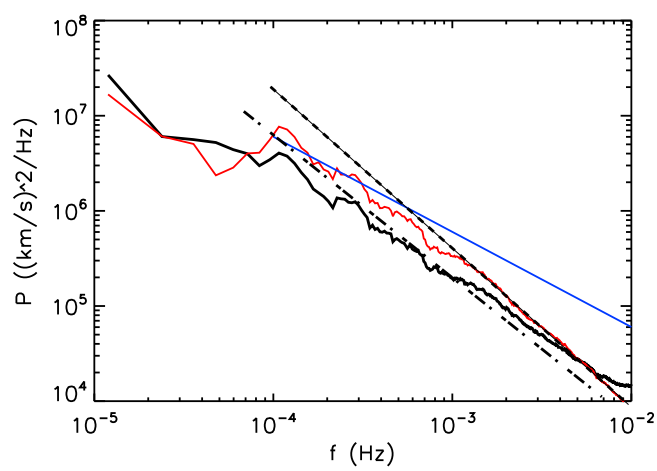

Figure 4. The power spectra of the velocity (black, thicker) and magnetic field (red) for an Alfvénic region ($\sigma_c \approx 0.8$) in Helios near 1 AU (day 24 of 1976). Reference spectra have indices as in Figure 3.

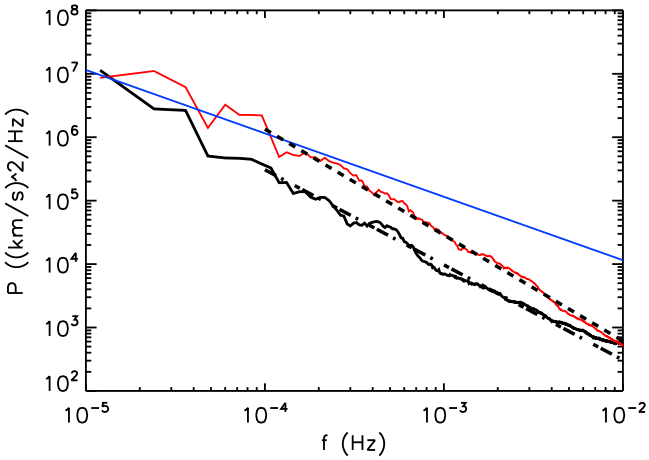

Figure 5. The power spectra of the velocity (black thicker) and magnetic field (red) for a relatively non-Alfvénic region ($\sigma_c \approx 0.2$) in Helios data near 1 AU (day 29 of 1976). Reference spectra have indices as in Figure 3.

Figure 4 is actually more complex, with a possible flattening at around 7×10^{-4} Hz in addition to that above 4×10^{-3} Hz.) The spectra in Figure 5, for a 340 km/s non-Alfvénic wind near 1 AU corresponding to the region in Figure 3 seen farther out but during an earlier solar rotation, are very similar to those in the study by *Podesta et al.* [2006].

[13] Ulysses data for much of the mission are shown in hour-averages in Figure 6. The bottom panel shows averages of the Alfvénicity over about a day, but since sectors are not taken into account (this being difficult when the relative fluctuations in the field are large), it is only in regions such as from 1993 to 1996 (minus the current sheet crossing in the middle) and from 2005 to 2007 that we can see clearly the natural evolution of the cross-helicity with distance since in these instances the spacecraft does not cross the heliospheric current sheet. The cross-helicity of typical outward propagating Alfvén waves changes sign, always being opposite to the polarity of the Sun's field (negative in an outward field sector), and this is taken into account in the examples below by choosing regions of one polarity or by multiplying the values of σ_c by -1 for one of the magnetic sectors before determining averages [see *Roberts et al.*, 1987]. Note (Figure 6) that the spacecraft crosses the ecliptic near perihelion and aphelion. We can see a very clear decrease of the Alfvénicity with distance in the outer heliosphere, as noted previously [e.g., *Roberts et al.*, 1987; *Neugebauer*, 2004; *B. E. Goldstein et al.*, 1995]. In the "noisy" regions where the value of σ_c is rapidly fluctuating, there will be some contribution from sector crossings, but, more significantly, the nonlinear evolution of the system produces sign reversals that are sometimes interpreted as the "production of inward propagating waves," although this is not the only possibility, and it may be that other modes or static structures may be involved [see *Bruno and Carbone*, 2005, section 3.2]. Near solar maximum, the solar wind is less ordered, and the tendency for oscillations in the cross-helicity is more common, thus leading to the "noisy" regions such as near perihelion in 2001. This data set was used here both for specific intervals and to perform a rough statistical study.

[14] We will proceed outward from the Sun using approximately 2000 h intervals. Figure 7 shows spectra from about

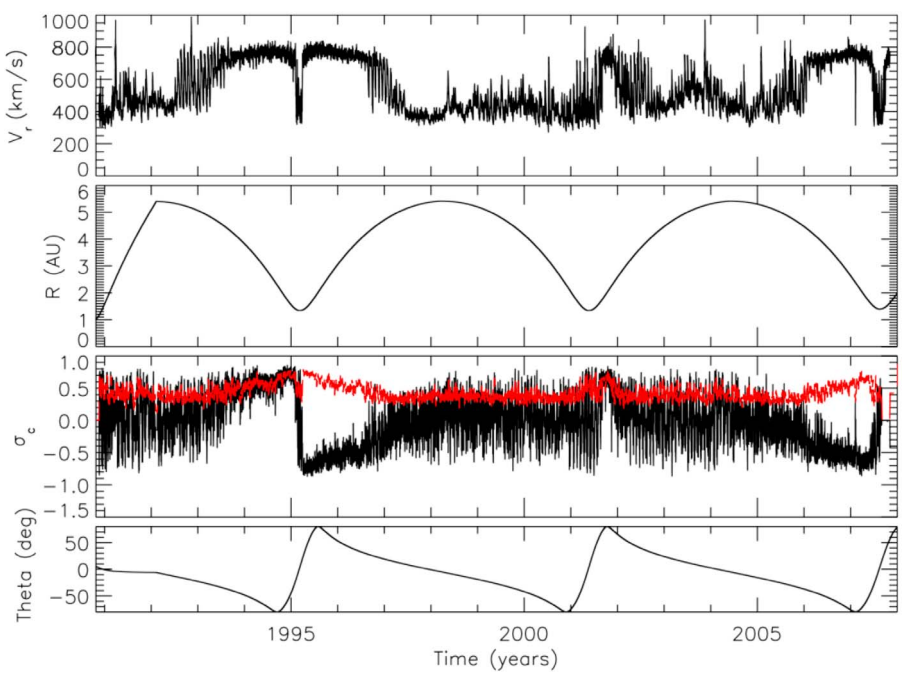

Figure 6. The hourly averaged solar wind speed, spacecraft radial distance from the Sun, Alfvénicity, and position angle of the spacecraft from the ecliptic for the Ulysses mission. The Alfvénicity is calculated a running 3 h correlation, and the result was smoothed with a 25 h window. The red line in that plot is the 4 day average absolute value of σ_c, and thus, it represents the upper limit of the average of σ_c in each region.

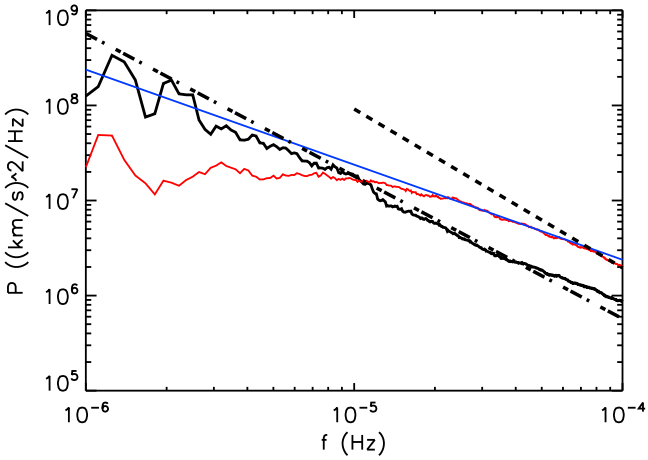

Figure 7. The power spectra of the velocity (black, thicker) and magnetic field (red) for the relatively Alfvénic ($\sigma_c \approx 0.7$), high-speed (about 750 km/s) region in Ulysses data from day 295 of 1994 to day 13 of 1995, near $R = 1.8$ AU. Reference spectra have indices as in Figure 3.

1.8 AU in highly Alfvénic wind; note that the Alfvénicity is less than it was in such streams closer in, but still quite high. At the frequencies accessible with hour averages, the spectra are similar to those in Figure 4 with the roll-over scale, to the extent it can be identified, at much lower frequency (now $\sim 10^{-5}$ Hz, and order of magnitude lower than in Figure 4). It is interesting to note that over this larger time interval, made possible by the passage of the spacecraft into relatively uniform wind away from the ecliptic, the ~ -1 spectral index region is apparent in the velocity at much lower frequencies than for the magnetic. The same behavior of the f^{-1} spectra is seen in Figure 8 in a somewhat less Alfvénic ($\sigma_c \approx 0.5$, amongst the highest values in the Ulysses mission for this distance) interval at 4.2 AU. The −5/3 slope is appearing in the magnetic field at the smallest scales, but the high-frequency velocity spectrum is still flatter than −3/2. Examination of the highest-resolution plasma data from Ulysses (4 min resolution, at times interpolated from 8 min) shows that the next decade in frequency continues the trend shown here, with a slope that is −3/2 or slightly flatter. In both Figures 7 and 8 instances, it might be that the brief region of steeper spectrum for the velocity between the highest and lowest regions (at around 10^{-5} Hz in Figure 7 and slightly lower in Figure 8) is a transition region in which the low frequencies are becoming involved in the cascade but not rapidly enough to keep up with the decay of the inertial range. (This is seen more prominently in Figure 9.) As in the inner heliosphere, as the cross-helicity becomes lower (as seen moving outward from the Sun in 1995 in Figure 6) there is a slightly greater dominance of the magnetic over the velocity energy at small scales. In this study the higher Alfvénicity regions never show a state in which the velocity spectrum becomes Kolmogoroff-like (with only one exception–see the discussion of Figure 12 below), although there is a slow evolution to near to the −3/2 case seen in the Podesta, et al. studies and a clear movement of the roll-over to flatter spectra to lower frequencies with decreasing Alfvénicity and increasing distance. A careful search might reveal a Voyager interval of high cross-helicity farther out than 5 AU, but the strong evolution in the ecliptic [*Roberts et al.*, 1987] and large data gaps will make this difficult.

[15] A major result of this paper is seen in Figures 9 and 10, both near 5 AU in low cross-helicity regions. Here we finally see further steepening of the velocity spectrum to −5/3. In Figure 9, near the ecliptic before the spacecraft was sent over the solar poles using a gravity assist from Jupiter, we see a particularly clear picture. In this case both of the reference steeper spectral lines have an index of −5/3, and they fit the high-frequency spectra very well. The spectra are parallel at small scales, but the break point to the −1 index occurs very clearly at a factor of 5 or so lower in frequency for the velocity, thus leading to the dominance of the magnetic field at small scales. In this sense, the velocity spectrum has become more "evolved," although it took it longer to attain the asymptotic Kolmogoroff slope. (See the related discussion by *Nicol et al.* [2009] of differences between velocity and magnetic spectra at large scales.) In Figure 10, the −3/2 slope is shown along with the −5/3; although the two are very similar, the observed spectrum is clearly above the steeper line at low frequency and below it at higher frequencies, and the −5/3 reference spectrum is an essentially perfect fit over the highest decade in frequency. At large scales, the kinetic energy of the fluctuations is still dominant. While the region shown in Figure 10 consists of three occurrences of alternating low-speed and high-speed streams, the same conclusions about the spectra are valid when only one high-speed region (≈700 km/s) is used in the analysis. The velocity spectrum has a–5/3 slope, in this case over much of the range from 10^{-6} to 10^{-4} Hz, and the magnetic spectrum has a roll-over between relatively flat and −5/3 slopes at about 10^{-5} Hz.

4. Radial Evolution: A Simple Statistical Study

[16] A simple statistical study provides a check on whether the above results are typical or due to sampling particular intervals. Figure 11 is based on spectra produced by an automated procedure in which spectra were calculated for

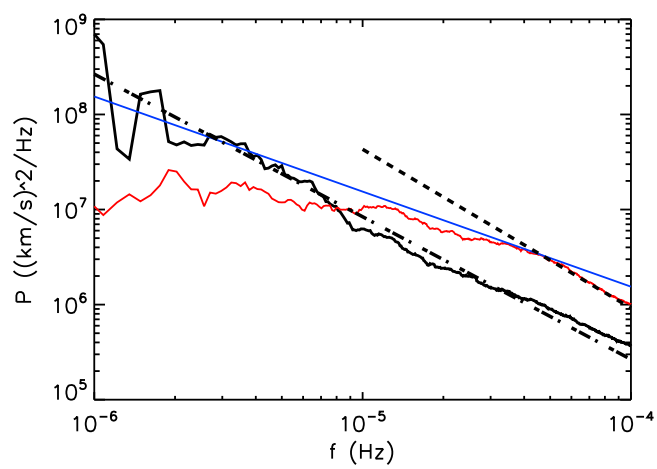

Figure 8. The power spectra of the velocity (black, thicker) and magnetic field (red) for the relatively Alfvénic ($\sigma_c \approx 0.5$, the highest in these data sets for this distance), high-speed (about 730 km/s) region in Ulysses data from days 243 to 329 of 1993, near $R = 4.2$ AU. Reference spectra have indices as in Figure 3.

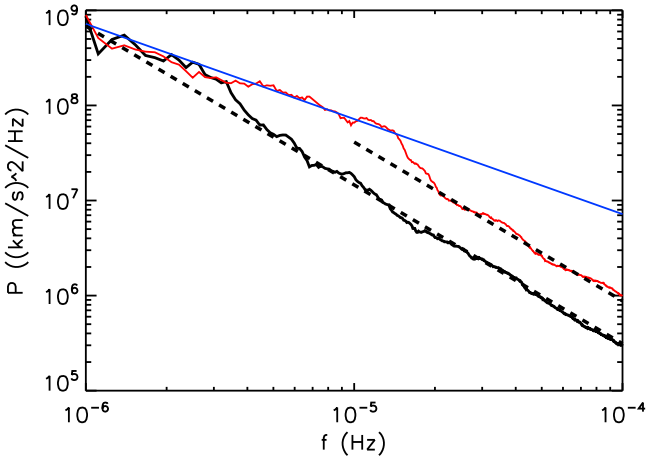

Figure 9. The power spectra of the velocity (black, thicker) and magnetic field (red) for the non-Alfvénic region (σ_c fluctuating near zero with a mean of 0.18, with sectors taken into account) in Ulysses data from day 308 of 1991 to day 26 of 1992, near $R = 4.8$ AU just before the Jupiter encounter, very near the ecliptic. The wind speed was about 500 km/s, with some higher speed deviations. Reference spectra have indices as in Figure 3, except that here both of the steeper spectral slopes are −5/3.

successive 3000 h intervals with a window sliding by 1000 h. This clearly mixes intervals of different speed, cross-helicity, and other properties, as was done in by *Podesta et al.* [2006], although with such long intervals of data it is perhaps reasonable to say that all "fluctuations," even stream structures, are just part of a statistical ensemble. To find the slopes of the resulting spectra, for the velocity it proved to be effective to simply choose two points a decade apart in frequency (at 1.1×10^{-5} and 1.1×10^{-4} Hz) to find the slope of the log-log plot.

[17] The resulting slopes for successive bins, along with the speed and distance at the center of that bin, are shown in Figure 11. The flat spectra are for highly Alfvénic polar wind. The −3/2 spectra found previously at 1 AU occur at the beginning of the mission, in first fast-latitude scans at about 1.3 AU at about bin 35, and to some degree in the other perihelion passes. The −5/3 values tend to occur farther out, although there are exceptions. These cases will be worth investigating in detail to see if they are associated with unusual characteristics of the flows. There is an interesting region, around bins 80–90, where the slope is substantially steeper than at other times. A detailed look at this showed that it is due to a −2 slope introduced over part of the range by strong jumps in the radial velocity, as has been previously studied by *Roberts and Goldstein* [1987]. In particular, these are not regions of radial field, and thus are unrelated to anisotropic turbulence theories (see, e.g., the discussion in the studies by *Horbury et al.* [2008] and *Tessein et al.* [2009]).

[18] A similar procedure for the magnetic field spectra is less successful because the roll-over to steeper slope occurs at various places within the upper decade in this case, making it difficult to get reliable slope values. Thus, this case is not shown. The magnetic evolution is well documented in the sources cited in the Introduction; the −5/3 spectrum appears at small scales at 0.3 AU, even in very Alfvénic regions, and is seen clearly, as also shown here, for all values of cross-helicity farther out.

5. Discussion and Conclusions

[19] This study documents the evolution of the solar wind velocity spectrum and thus provides evidence that the "3/2 spectrum" for velocity fluctuations previously noted at 1 AU is a transient effect, and that a Kolmogoroff −5/3 spectrum is the asymptotic state of the turbulence for both the velocity and magnetic fields. The velocity spectral evolution from 1 to 5.4 AU is summarized in Figure 12, which plots the slopes and distances of Figure 11 against each other. The plusses are for high speed (above 675 km/s), and they show that the spectra in fast winds have clearly changed with distance, with no cases of −3/2 slope beyond 4.5 AU, the values being fairly uniformly distributed around the −5/3 value. The average slope (with the error in the mean $\propto 1/sqrt(N)$ where N is the number of points) for $R > 4.5$ AU is -1.684 ± 0.008. When the points are restricted to speeds greater than 675 km/s, the average is -1.684 ± 0.025, and for speeds less than 475 km/s, the average is -1.667 ± 0.011. The lack of velocity dependence for the spectral slope at larger distances is similar to the finding of *Tessein et al.* [2009, Figure 3] at 1 AU, where the mean slopes for slow and fast wind have very similar distributions, quite different from those for the magnetic field, but in this case centered on values near −1.5, and perhaps closer to −1.35 in the slow wind case. In Figure 12, the points that are within 20° of the ecliptic are in blue squares. There are few points near 1 AU, but most of them out to 2 AU have slopes near −1.5, consistent with Tessein et al. Since Ulysses travels more slowly at aphelion, there are many more points

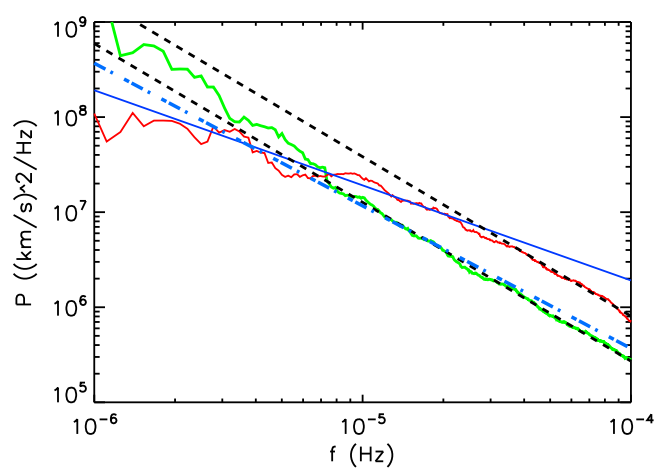

Figure 10. The power spectra of the velocity (green for clarity in this case, and thicker as before) and magnetic field (red) for the relatively non-Alfvénic region (σ_c fluctuating near zero with only one magnetic sector throughout, and with the average of $\sigma_c = -0.25$) in Ulysses data from day 315 of 1996 to day 32 of 1997, near $R = 4.7$ AU. Reference spectra have indices as in Figure 3, with an additional −5/3 included for the magnetic spectrum. This region consists of alternating high-speed and low-speed wind, but the same results for spectra are obtained with a high-speed wind subset. Sighting along the lines helps to sort out the very close lines for the velocity spectrum.

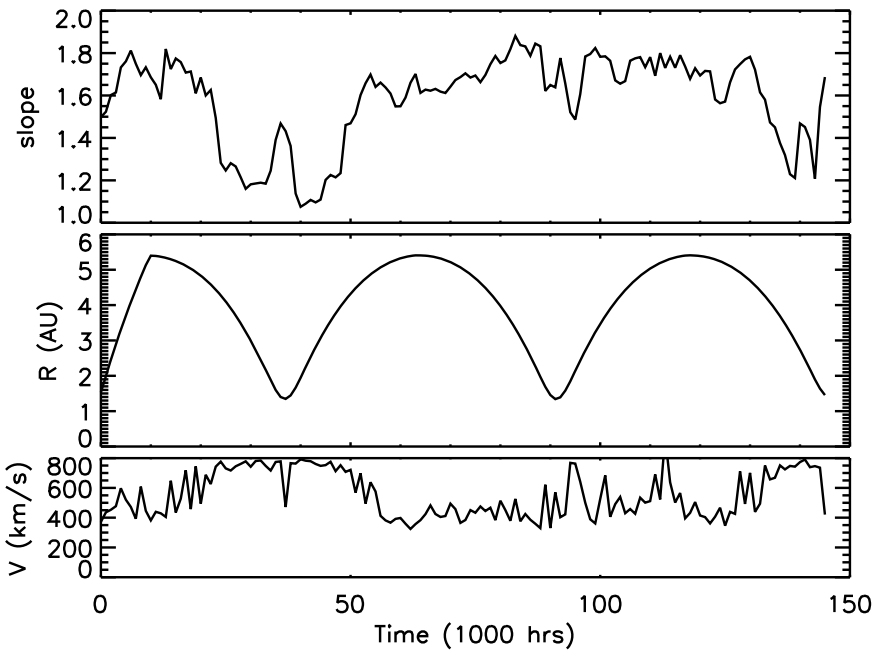

Figure 11. The slopes (×−1) of the highest decade of the velocity spectra in the Ulysses data, binned in 3000 h intervals sliding by 1000 h; the value on the *x* axis is thus the bin number. Also included are the radial velocity measured by Ulysses at the middle of the bin and the spacecraft distance from the Sun at that time. The dates along the *x* axis may be found by direct comparison of speed and distance with the same traces in Figure 6.

near the ecliptic beyond 4.5 AU, and these all enter into the averages stated above.

[20] A wide variety of conditions closer to the Sun are gradually replaced by a convergence to a Kolmogoroff-like spectrum by about 4.5 AU. It is also true that by that distance, the Alfvénicity of the fluctuations has decreased to fairly low values for all the solar wind conditions that Ulysses sampled, as indicated by the lack of symbols around the points: triangles are around points with the magnitude of $\sigma_c > 0.5$ and diamonds are around points with $0.5 > \sigma_c > 0.33$ as determined by averages of the cross-helicity times the sign of the radial magnetic field. (The results for identifying higher cross-helicity regions are not sensitive to the detailed criterion used.) It is suggestive that between 1.5 and 2.4 as well as between 3.7 and 4.4 AU the high-speed points with flatter spectra have higher cross-helicity than those with steeper spectra, consistent with the view that high Alfvénicity slows the evolution. Unfortunately, we have found no cases of highly Alfvénic flows at greater distances than the 4.2 AU of Figure 8, and thus we do not know how the velocity spectrum evolves at very large distances from the Sun in those conditions. There is a general tendency for the cross-helicity to decrease with distance, but the decline to nearly random values beyond 4.5 AU may be an artifact of the orbit of the spacecraft. The spacecraft necessarily comes closer to the ecliptic at large distances, so it may be that the higher cross-helicity values would be found in more uniform polar flows, but these have not been found in the current data. It may be possible to find shorter or higher-resolution intervals of high cross-helicity at large distances, but this is left to another study. According to the evidence here, the main velocity spectral evolution for high cross-helicity is in the inner heliosphere. At the closest distances to the Sun, the generally quite flat velocity spectrum is equipartitioned in energy with the magnetic field except at higher frequencies where it dominates (Figure 2). This spectrum becomes somewhat

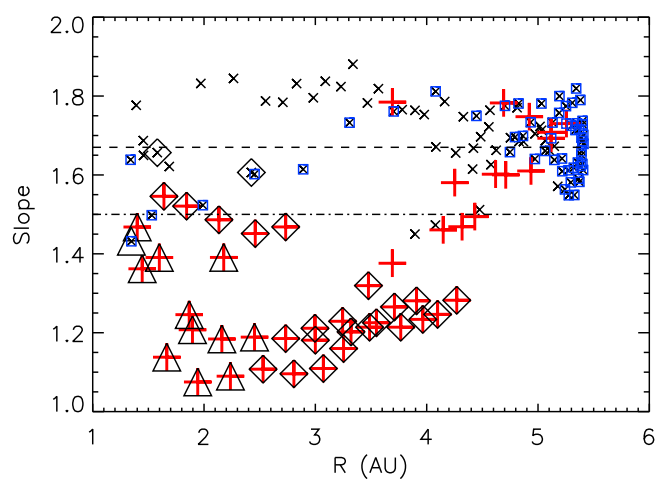

Figure 12. The slopes of Figure 11, plotted as a function of radial distance. Points for which the speed is greater than 675 km/s are shown as red plusses, with all other points shown as black crosses. Points with a (signed) Alfvénicity between 0.33 and 0.5 are enclosed in black diamonds, those with Alfvénicity greater than 0.5 are enclosed in black triangles, and points that are within 20° of the ecliptic are shown in blue squares. Horizontal lines are drawn at slope values of −1.5 and −1.67. There is a convergence with radial distance to slopes near −5/3 for both slow and fast wind.

lower in energy compared to the magnetic by 1 AU (Figure 4), and the slope becomes steeper but not −5/3. After that (Figures 7 and 8), the ratio of fluctuating velocity to magnetic field decreases somewhat to around the typical value of 1/2, but otherwise the spectrum does not change much. In Figure 12, the diamonds show regions of moderately-high cross-helicity, only one of which exhibits a −5/3 slope, whereas bins with $\sigma_c > 0.5$ consistently have slopes flatter than −3/2 with only one close to that value. There are no diamonds beyond 4.2 AU. These observations are consistent with the viewpoint, discussed in the introduction, that highly Alfvénic regions have their nonlinear evolution suppressed.

[21] For any cross-helicity, the difference in the magnetic evolution is that it attains a −5/3 spectrum at small scales much earlier than the velocity fluctuations (even by 0.3 AU, especially for low Alfvénicity), but this spectrum does not "eat into" the larger scales as rapidly as the velocity spectrum does. Figure 9 shows this especially clearly. In many cases shown here, the roll-over in the velocity spectrum is not apparent in the frequency range displayed, although it must occur at some very low frequency. The difference in the evolution of the roll-over point in the spectra provides a new view of the common observation that magnetic fluctuations are very frequently more energetic than the velocity at small scales: it may be due to a more effective cascade of the velocity. Still, it is not clear why the velocity takes so long to achieve the final asymptotic slope (assuming there is no further evolution farther out), nor is it clear why the magnetic field retains a roughly −1 slope at intermediate scales longer than the velocity does. These questions may only be answered by simulations or theory, but further work along the present lines will clarify the phenomenology and thus guide the simulations.

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References

Bavassano, B., M. Dobrowolny, F. Mariani, and N. F. Ness (1982), Radial evolution of power spectra of interplanetary Alfvenic turbulence, *J. Geophys. Res.*, *87*(A5), 3617, doi:10.1029/JA087iA05p03617.

Belcher, J. W., and L. Davis Jr. (1971), Large-amplitude Alfvén waves in the interplanetary medium, 2, *J. Geophys. Res.*, *76*(16), 3534, doi:10.1029/JA076i016p03534.

Bruno, R., and V. Carbone (2005), The solar wind as a turbulence laboratory, *Living Rev. Sol. Phys.*, *2*, 4.

Coles, W. A., W. Liu, J. K. Harmon, and C. L. Martin (1991), The solar wind density spectrum near the Sun—Results from Voyager radio measurements, *J. Geophys. Res.*, *96*(A2), 1745, doi:10.1029/90JA01991.

Goldstein, B. E., E. J. Smith, A. Balogh, T. S. Horbury, M. L. Goldstein, and D. A. Roberts (1995), Properties of magnetohydrodynamic turbulence in the solar wind as observed by Ulysses at high heliographic latitudes, *Geophys. Res. Lett.*, *22*(23), 3393, doi:10.1029/95GL03183.

Goldstein, M. L., D. A. Roberts, and W. H. Matthaeus (1995), Magnetohydrodynamic turbulence in the solar wind, *Annu. Rev. Astron. Astrophys.*, *33*, 283, doi:10.1146/annurev.aa.33.090195.001435.

Grappin, R., A. Mangeney, and E. Marsch (1990), On the origin of solar wind MHD turbulence—HELIOS data revisited, *J. Geophys. Res.*, *95*(A6), 8197, doi:10.1029/JA095iA06p08197.

Horbury, T. S., and A. Balogh (2001), Evolution of magnetic field fluctuations in high-speed solar wind streams: Ulysses and Helios observations, *J. Geophys. Res.*, *106*(A8), 15,929, doi:10.1029/2000JA000108.

Horbury, T. S., M. Forman, and S. Oughton (2008), Anisotropic scaling of magnetohydrodynamic turbulence, *Phys. Rev. Lett.*, *101*, 175005, doi:10.1103/PhysRevLett.101.175005.

Kolmogorov, A. (1941), The local structure of turbulence in incompressible viscous fluid for very large Reynolds' numbers, *C. R. Acad. Sci. URSS*, *30*, 201.

Kraichnan, R. H. (1965), Inertial-range spectrum of hydromagnetic turbulence, *Phys. Fluids*, *8*, 1385, doi:10.1063/1.1761412.

Marsch, E., and C.-Y. Tu (1990a), On the radial evolution of MHD turbulence in the inner heliosphere, *J. Geophys. Res.*, *95*(A6), 8211, doi:10.1029/JA095iA06p08211.

Marsch, E., and C.-Y. Tu (1990b), Spectral and spatial evolution of compressible turbulence in the inner solar wind, *J. Geophys. Res.*, *95*(A8), 11,945, doi:10.1029/JA095iA08p11945.

Matthaeus, W. H., and M. L. Goldstein (1982), Measurement of the rugged invariants of magnetohydrodynamic turbulence in the solar wind, *J. Geophys. Res.*, *87*(A8), 6011, doi:10.1029/JA087iA08p06011.

Neugebauer, M. (2004), Anisotropy and Alfvénicity of hourly fluctuations in the fast polar solar wind, *J. Geophys. Res.*, *109*, A02101, doi:10.1029/2003JA009947.

Nicol, R. M., S. C. Chapman, and R. O. Dendy (2009), Quantifying the anisotropy and solar cycle dependence of "1/f" solar wind fluctuations observed by advanced composition explorer, *Astrophys. J.*, *703*, 2138, doi:10.1088/0004-637X/703/2/2138.

Podesta, J. J., D. A. Roberts, and M. L. Goldstein (2006), Power spectrum of small-scale turbulent velocity fluctuations in the solar wind, *J. Geophys. Res.*, *111*, A10109, doi:10.1029/2006JA011834.

Podesta, J. J., D. A. Roberts, and M. L. Goldstein (2007), Spectral exponents of kinetic and magnetic energy spectra in solar wind turbulence, *Astrophys. J.*, *664*, 543, doi:10.1086/519211.

Roberts, D. A. (1989), Interplanetary observational constraints on Alfvén wave acceleration of the solar wind, *J. Geophys. Res.*, *94*(A6), 6899, doi:10.1029/JA094iA06p06899.

Roberts, D. A. (2010), Demonstrations that the solar wind is not accelerated by waves or turbulence, *Astrophys. J.*, *711*, 1044, doi:10.1088/0004-637X/711/2/1044.

Roberts, D. A., and M. L. Goldstein (1987), Spectral signatures of jumps and turbulence in interplanetary speed and magnetic field data, *J. Geophys. Res.*, *92*(A9), 10,105, doi:10.1029/JA092iA09p10105.

Roberts, D. A., M. L. Goldstein, L. W. Klein, and W. H. Matthaeus (1987), Origin and evolution of fluctuations in the solar wind—HELIOS observations and Helios-Voyager comparisons, *J. Geophys. Res.*, *92*(A11), 12,023, doi:10.1029/JA092iA11p12023.

Tessein, J. A., C. W. Smith, B. T. MacBride, W. H. Matthaeus, M. A. Forman, and J. E. Borovsky (2009), Spectral indices for multi-dimensional interplanetary turbulence at 1 AU *Astrophys. J.*, *692*, 684, doi:10.1088/0004-637X/692/1/684.

Verma, M. K., D. A. Roberts, M. L. Goldstein, S. Ghosh, and W. T. Stribling (1996), A numerical study of the nonlinear cascade of energy in magnetohydrodynamic turbulence, *J. Geophys. Res.*, *101*(A10), 21,619, doi:10.1029/96JA01773.

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