Beam attenuation coefficient retrieval by inversion of airborne lidar-induced chromophoric dissolved organic matter fluorescence. I. Theory

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It is shown that the oceanic beam attenuation coefficient can be retrieved from airborne laser-induced and depth-resolved chromophoric dissolved organic matter (CDOM) fluorescence. The radiative transfer equation (RTE) retrieval methodology does not require a laser beam spread function model since two CDOM fluorescence bands are used in conjunction with a beam attenuation spectral model, is self-normalizing since the CDOM absorption coefficient and laser beam irradiance are common to both fluorescence observational channels, and is enabled by the known isotropic phase function for CDOM fluorescence. Although this RTE analytical inversion theory is exact, the retrieval uncertainty is reduced by configuring the proposed lidar in the multiple-field-of-view beam attenuation mode to significantly diminish observation of multiple scattering. The theory can be applied over wide regions of the ocean's continental margins, estuaries, lakes, and rivers that are known to have sufficient CDOM. © 2006 Optical Society of America

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1. Introduction

Beam attenuation coefficients are a fundamental measurement in ocean optics and this is especially true since it contains all the component absorption and scattering.[1] Also, beam attenuation measurements have recently been closely linked to a major component of the global carbon cycle: particulate organic carbon (POC).[2,3] Since no airborne or satellite retrieval algorithms for beam attenuation or POC are presently available, efforts are needed to address this void. Once the airborne lidar beam attenuation is in place, then satellite algorithm development can be assisted by lidar through use of simultaneous airborne passive (solar) ocean color spectroradiometry.[4–9] Also, the same airborne methodology can later provide satellite algorithm validation by airborne underflights of ocean color satellite sensors.

Chromophoric dissolved organic matter (CDOM) is also an important carbon-containing constituent of the ocean that is not only a valuable active and passive tracer of the physical forcing of the ocean[6,7,10–12] but, as described herein, is a valuable constituent that allows retrieval of oceanic inherent optical properties. In the past one of the primary uses of lidar-induced CDOM fluorescence was for empirical determination of the CDOM absorption coefficient. The present radiative transfer equation (RTE) inversion effort is significantly less empirical and addresses the application of CDOM fluorescence to retrieval of the beam attenuation coefficient.

The theory derived herein is meant to complement a beam attenuation coefficient concept that uses the single-channel OH-stretch water Raman backscattering.[13] Compared with water Raman scattering, use of CDOM fluorescence for beam attenuation retrieval is more complex. For example, water Raman scattering is prompt with essentially no temporal or pulse broadening and has only a small temperature variability that can be ignored for beam attenuation retrievals; it also has a constant scattering coefficient. On the other hand, CDOM fluorescence has a temporal decay lifetime that can reach ~9 ns, has a regionally variable absorption coefficient (and associated fluorescence) that is dependent on assorted concentrations of molecularly variable chromophores and on the amount of solar irradiance and resulting photodegradation, and has a significant negative correlation with salinity[14] especially in coastal regions having low salinity runoff. As described in this paper, some of these issues are addressed through use of two CDOM spec-

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tral bands. (For remote Raman spectroscopy, a two-band option is not presently available since only the OH-stretch Raman band is strong enough for remote sensing.) Also, because it is prompt, the water Raman can be used to assist in the correction and removal of the temporal decay of CDOM fluorescence from the desired temporal decay (i.e., radiance variability with depth) caused by inherent water column attenuation. Concurrently, the lidar receiver electronic temporal decay characteristics can be removed. Thus the water Raman can simultaneously be applied to both the CDOM fluorescence decay and the receiver electronic pulse broadening.

The beam attenuation theory herein can be applied only in regions having CDOM such as continental shelves, estuaries, lakes, and rivers. However, these regions occupy considerable portions of the world's oceans. CDOM is sometimes called yellow substance,[5] and for simplification of the mathematical equations, the superscript or subscript Y is used in place of CDOM for simplification.

2. Chromophoric Dissolved Organic Matter Fluorescence Lidar for Beam Attenuation Retrieval

The RTE does not include and therefore will not accommodate observation of multiple scattering (except under special applications such as Monte Carlo and invariant embedding methods[1]). Thus the lidar should be configured to minimize the observation of multiple scattering. This is largely accomplished by a laser transmitter having both a small beam diameter and a narrow beam divergence and a multiple-field-of-view (MFOV) receiver to achieve what is known as the beam attenuation mode (BAM).[13] Some of these features of the BAM[13] are summarized below.

A. Laser

Although other laser sources could be used, a frequency-tripled 1064 nm Nd:YAG laser operating at 355 nm is chosen for the airborne beam attenuation retrieval methodology herein.

An important characteristic of 355 nm excitation of natural waters is the rather broad CDOM fluorescence spectrum that spans ~355–600 nm. (For an example of the CDOM spectrum,[6] consult Fig. 3 in Ref. 6.) The CDOM absorption spectrum is also rather broad[15] and this facilitates stimulation of fluorescence over wide regions of the UV and visible excitation spectrum by use of other laser types if desired.

For the 355 nm laser, undesired phytoplankton inelastic emissions falling within the ~355–600 nm CDOM spectral band (such as chlorophyll and phycoerythrin fluorescence) have not been reported and thus are not expected to need removal or correction. The chlorophyll a fluorescence band at ~683 nm is excited by 355 nm,[16] but can easily be avoided. On the other hand, the ~11 nm wide inelastic water Raman band centered at ~402 nm should not be used to provide the required CDOM signals unless robust Raman correction or removal procedures are used.[13]

As will be seen in Appendix A, it is desirable to utilize a laser having a temporal width corresponding to a Dirac delta function. However, typically available with pulse widths of ~3–12 ns,[17] the frequency-tripled Nd:YAG 355 nm laser is suitable for this beam attenuation application.

B. Laser Receiver

The principal feature of the lidar fluorosensor is the MFOV receiver. [For example, a straightforward economical configuration might use a stepping motor to sequentially position individual field-of-view (FOV) elements in the optical path with accompanying degradation of the horizontal spatial resolution resulting from airborne motion. Similarly, a multiple concentric ring optical fiber bundle would mitigate the spatial resolution degradation by use of a multiple, more expensive spectrometer configuration.] The MFOV allows extrapolation to 0° FOV to enhance or ensure the BAM and minimizes observation of multiple scatter. Additionally, the individual CDOM spectral bands (to be positioned within the broad CDOM fluorescence emission) should be sufficiently wide to provide adequate signal but they should be sufficiently narrow to define the spectral location and to avoid unwanted inelastic emissions if any.

In Appendix A, a required quantity in the RTE is the depth-resolved backscattered radiance $\mathrm{d}L(z; \theta, \phi; \lambda_F)/\mathrm{d}z$, i.e., the actual radiance variation with depth. However, the pulsed lidar measures the temporal waveform $\mathrm{d}L(t = 2zm/v; \theta, \phi; \lambda_F)/\mathrm{d}z$, where v is the velocity of light, m is the index of refraction of seawater, and t is the two-way travel time to depth z. This lidar measurement includes (a) the finite-width laser pulse convolved with (b) the sea state or height or temporal distribution during double passage through the sea surface; (c) the finite bandwidth of the receiver's detector, amplifier, and pulse digitizer (i.e., system response or system time dispersion); (d) the CDOM fluorescence lifetime; and (e) the desired radiance decay $\mathrm{d}L(z; \theta, \phi; \lambda_F)/\mathrm{d}z$ to be used within the RTE inversion. For example,

$$\frac{\mathrm{d}L(t = 2zm/v; \theta, \phi; \lambda_F)}{\mathrm{d}z} = P \otimes S \otimes W \otimes F \otimes \frac{\mathrm{d}L(z; \theta, \phi; \lambda_F)}{\mathrm{d}z}, \quad (1)$$

where $\otimes$ denotes the convolution operation, and P, S, W, and F are models or measurements that describe the laser pulse, sea state, receiver bandwidth, and CDOM fluorescence lifetime, respectively. The models for P, S, and, R and the resulting deconvolutions are rather straightforward.[18] Thus the required $\mathrm{d}L(z; \theta, \phi; \lambda_F)/\mathrm{d}z$ is obtainable from the lidar-measured $\mathrm{d}L(t = 2zm/v; \theta, \phi; \lambda_F)/\mathrm{d}z$ upon deconvolution or removal of the P, S, W, and F characteristics. Models and the solution of this type of equation in a lidar situation have been discussed in detail[18] except for the fluorescence lifetime considerations. A fluorescence lifetime model is

$$F = F_0 \exp\left(\frac{-1}{\tau}\right), \tag{2}$$

where F_0 is the peak CDOM fluorescence in the CDOM channel and τ is the known CDOM fluorescence lifetime for the oceanographic region under study. For example, the time required for the peak fluorescence to decline to $1/e$ defines the fluorescence lifetime τ. However, another option is available for estimation of the combined convolution $P \otimes S \otimes W$ using the elastic backscatter at wavelength λ_L and/or the CDOM-free water Raman backscatter at wavelength λ_R with appropriate corrections for use at the CDOM emission wavelength at λ^Y. Alternately, for use in the deconvolution calculation, the instrument temporal response $P \otimes W$ component can be obtained by viewing an impenetrable nonfluorescent flat target prior to airborne flight[18] and measuring the temporal response (preferably using a pulsed laser at wavelengths centered within the Raman and CDOM channels). Of course, if the laser or Raman wavelengths are used for these components, they must be wavelength corrected for their application at the CDOM observational wavelengths.

In short, the observational CDOM channels must have their intrinsic hardware temporal response, and other undesirable responses removed for the measured time decay to exactly represent only the time decay attributable to the in-water beam attenuation.[18,19] As shown in Section 3, depth-resolved waveform data at the two CDOM wavelengths are used to derive the quantities needed to retrieve the beam attenuation inherent optical properties in the upper surface layer.

In addition to possible use in the deconvolution procedures, an elastic-backscattering channel at 355 nm (Ref. 17) is recommended to initiate the receiver time digitizer measurements for aircraft altitude and depth-resolved measurements for consistency checks and possible validation.

C. Beam Attenuation Mode

A good example of an initial or starting configuration for implementation of a CDOM fluorescence lidar having MFOV and BAM characteristics is the airborne oceanographic lidar.[4,5,8,9,17] The CDOM fluorescence lidar may be configured for the BAM[13] as follows: (a) the monochromatic laser transmitter is unpolarized and highly collimated to a small diameter and small diffraction-limited beam divergence angle, (b) the polarization-insensitive receiver has a very narrow FOV intended to provide a viewed surface target spot diameter that is much smaller than the mean free path of the in-water photons,[20] and (c) both transmitter and receiver are in coaxial alignment. In this BAM, the center axis of the diverging laser beam source is aligned vertically downward along the $+z$ axis. With the very narrow FOV lidar receiver being coaxial with the laser transmitter, backscattered photons are viewed as they travel upward to the receiver vertically in the $-z$ direction. To provide additional confidence in the BAM configuration, the results of Gordon[20] are used to suggest that the BAM be implemented with a MFOV receiver, which is fairly common in atmospheric lidars[21–24] and only recently suggested for oceanic lidar use.[25] The likelihood of multiple scattering is characterized by the radius[20] $\mathcal{R}$ of the spot viewed on the ocean surface divided by the mean free path between photon collisions (given by $1/c$) or $c\mathcal{R}$. Referring to Figs. 3 and 4 in Ref. 20, it is seen that as $c\mathcal{R} \Rightarrow 0$, then $k/c \Rightarrow 1$ independent of the elastic-scattering phase function (where k is the lidar attenuation coefficient). This strongly suggests that a lidar receiver having approximately three to four discrete FOVs would allow extrapolation to null a FOV corresponding to a dependable BAM. For example, although a single FOV could be used, these data[20] suggest that a MFOV configuration would be more robust. Each individual FOV could be sequentially stepped into the receiver to provide the required MFOV data. To help maintain the dynamic range of the lidar receiver, the individual FOVs could be implemented as clear annular rings of ever-increasing radii.

It is important to point out that solar and sky irradiance scattered both elastically and inelastically into the lidar-receiving telescope are not measured since the receiver photomultiplier tube detectors are ac coupled to avoid such dc background radiation.[8,9,13] This dc background is further rejected by use of narrowband interference filters centered on the desired spectral bands and, as mentioned above, a very small FOV receiver to observe the small laser footprint on the ocean surface. These latter two lidar properties also help maintain the phototubes within their linear operating range, especially during daylight operation. Accordingly, no solar and sky irradiance need be considered in this theoretical development. In fact the beam attenuation inherent optical properties retrieval method described herein can be applied in total darkness or during daylight.

3. Theory

From Appendix A, relation (A8), the RTE for a BAM lidar-viewing laser-induced fluorescence at wavelength λ_F is

$$\begin{aligned} -\frac{\mathrm{d}L(z;\,\theta,\,\phi;\,\lambda_F)}{\mathrm{d}z} \cong & -c(z;\,\lambda_F)L(z;\,\theta,\,\phi;\,\lambda_F) \\ & + a^F(z,\,\lambda_L{}')f^F(\lambda_L{}' \to \lambda_F) \\ & \times \tilde{\beta}^F(\psi)\int_0^{2\pi}\int_0^{\pi} L(z;\,\theta',\,\phi';\,\lambda_L{}') \\ & \times \sin\theta'\,\mathrm{d}\theta'\mathrm{d}\phi'. \end{aligned} \tag{A8}$$

Note in Eq. (A8) that the CDOM absorption coefficient at the laser wavelength $a^F(z, \lambda_L{}')$ is not known. Thus one cannot solve for the beam attenuation $c(z, \lambda)$ by simply inserting a laser beam spread function (BSF) model for the laser radiance $L(z; \theta', \phi'; \lambda_L{}')$ and then performing the integral. (Such a single-channel methodology can be used to retrieve

the beam attenuation when the water Raman band[13] is used since the Raman-scattering coefficient is known.) However, use of two CDOM observational fluorescence bands will allow retrieval of the beam attenuation coefficient. For 355 nm excitation, since the CDOM fluorescence emission band spans ~355–600 nm, then this span easily allows the two required observational band segments to be identified and established within this broad emission spectrum. The water Raman band centered at ~402 nm should be avoided as a location of one of the CDOM spectral bands. The peak CDOM fluorescence emission at ~450 nm region[6] can be used if the second chosen band provides sufficient spectral separation from it and adequate signal strength.

Writing Eq. (A8) for each of the two chosen CDOM fluorescence emission wavelengths λ_{Y1} and λ_{Y2} yields the required RTE equations:

$$-\frac{\mathrm{d}L(z;\,\theta,\,\phi;\,\lambda_{Y1})}{\mathrm{d}z} \cong -c(z;\,\lambda_{Y1})L(z;\,\theta,\,\phi;\,\lambda_{Y1}) + a^Y(z,\,\lambda_L')f^Y(\lambda_L' \to \lambda_{Y1}) \times \tilde{\beta}^Y(\psi)\int_0^{2\pi}\int_0^{\pi} L(z;\,\theta',\,\phi';\,\lambda_L') \times \sin\theta'\,\mathrm{d}\theta'\mathrm{d}\phi', \tag{3}$$

$$-\frac{\mathrm{d}L(z;\,\theta,\,\phi;\,\lambda_{Y2})}{\mathrm{d}z} \cong -c(z;\,\lambda_{Y2})L(z;\,\theta,\,\phi;\,\lambda_{Y2}) + a^Y(z,\,\lambda_L')f^Y(\lambda_L' \to \lambda_{Y2}) \times \tilde{\beta}^Y(\psi)\int_0^{2\pi}\int_0^{\pi} L(z;\,\theta',\,\phi';\,\lambda_L') \times \sin\theta'\,\mathrm{d}\theta'\mathrm{d}\phi', \tag{4}$$

where $f^Y(\lambda_L' \to \lambda_{Y1})$ and $f^Y(\lambda_L' \to \lambda_{Y2})$ are the lidar receiver bandpass segments contained within the overall $f^Y(\lambda_L' \to \lambda_Y)$ 355–600 nm redistribution spectrum.

There are three unknowns in relations (3) and (4): $c(z, \lambda_{Y1})$, $c(z, \lambda_{Y2})$, and $G \equiv a^Y(\lambda_L')\,\beta^Y(\psi)\int_0^{2\pi}\int_0^{\pi} L(z;\,\theta',\,\phi';\,\lambda_L')\sin\theta'\mathrm{d}\theta'\mathrm{d}\phi'$, the latter unknown being common to both relations (3) and (4) (and for convenience the known isotropic phase function $\tilde{\beta}^Y(\psi) = 1/4\pi$ is also being carried within G). Note that G (containing the laser beam radiance and CDOM absorption) is common to both relations (3) and (4) and suggests self-normalization since it is eliminated during the simultaneous solution of relations (3) and (4) and therefore does not appear in the resulting beam attenuation retrieval equations below. For example, any change in the laser transmit power incident upon the ocean due to variations in laser output, aircraft altitude variation, atmospheric transmission, etc. are not embedded in the beam attenuation retrieval since both CDOM bands within the entire CDOM fluorescence spectrum incur the same percentage change. Similarly, variations in the CDOM absorption coefficient (a^Y, that is also a component of G) do not affect the beam attenuation coefficient retrieval. The signal amplitudes of the two observational CDOM fluorescence bands increase (or decrease) in unison as a result of changes in the laser radiance at the sea surface and/or the CDOM absorption coefficient, and these variations are not propagated into the beam attenuation retrieval. Also, if salinity changes affect the entire CDOM spectrum equally, then these salinity-induced variations are also similarly not impressed within the beam attenuation coefficient retrieval.

A beam attenuation coefficient spectral model supplies an additional or third equation and allows solution for the three unknowns listed above. A beam attenuation coefficient spectral model is[26,27]

$$c(\lambda,\,z) - c_w(\lambda,\,z) = [c(\lambda_{\mathrm{ref}},\,z) - c_w(\lambda_{\mathrm{ref}},\,z)](A + B\lambda), \tag{5}$$

where λ_{ref} is a reference wavelength [e.g., 490 nm (Ref. 26)], A and B are known constants, and $c_w(\lambda, z)$ is the beam attenuation of seawater. [Only the constituent beam attenuation at the reference wavelength $c(\lambda_{\mathrm{ref}}, z) - c_w(\lambda_{\mathrm{ref}}, z)$ is required since it will then allow calculation of the beam attenuation at all wavelengths, $c(\lambda, z) - c_w(\lambda, z)$. The inversion results may be improved by adopting constants A and B for the specific oceanic province being studied.] The needed $c(\lambda_{\mathrm{ref}}, z) - c_w(\lambda_{\mathrm{ref}}, z)$ is easily obtained by the following steps: (a) evaluate Eq. (5) at the first CDOM band centered at λ_{Y1} and substitute the resulting $c(\lambda_{Y1}, z)$ into relation (3); (b) similarly evaluate Eq. (5) at the second CDOM band centered at λ_{Y2} and substitute the resulting $c(\lambda_{Y2}, z)$ into relation (4); (c) eliminate G between the resulting equations derived in steps (a) and (b); and (d) solve for $c(\lambda_{\mathrm{ref}}, z) - c_w(\lambda_{\mathrm{ref}}, z)$ to yield

$$c(\lambda_{\mathrm{ref}},\,z) - c_w(\lambda_{\mathrm{ref}},\,z) = \frac{f_1(c_{w2}L_2 - D_2) - f_2(c_{w1}L_1 - D_1)}{l_1L_1f_2 - l_2L_2f_1}, \tag{6}$$

where $f_i = f^Y(\lambda_L' \to \lambda_{Yi})$, $L_i = L(z;\,\theta,\,\phi;\,\lambda_{Yi})$, $D_i = [\mathrm{d}L(z;\,\theta,\,\phi;\,\lambda_{Yi})]/\mathrm{d}z$, $l_i = (A + B\lambda_i)$, and $c_{wi} = c_w(\lambda_i, z)$ where $i = 1, 2$. Note that the fluorescence redistribution ratio for the two bands $p = f_1/f_2$ is a known constant for a single laser excitation wavelength and this allows further simplification:

$$c(\lambda_{\mathrm{ref}},\,z) - c_w(\lambda_{\mathrm{ref}},\,z) = \frac{p(c_{w2}L_2 - D_2) - (c_{w1}L_1 - D_1)}{l_1L_1 - pl_2L_2}. \tag{7}$$

In lieu of published[1] distribution functions, to determine a more accurate value of p for the water mass or region being studied, a laboratory 355 nm excited CDOM fluorescence spectrum of a water sample can be used.

Using Eq. (5) one can now obtain the desired $c(\lambda, z)$ at any wavelength. Again, one must choose the wavelengths λ_{Y1} and λ_{Y2} within the valid wavelength

range for the beam attenuation spectral model and within the laser-induced CDOM fluorescence spectrum to have sufficient spectral separation and signal strength to yield an effective retrieval. Since $c(z, \lambda)$ is now known, if desired, G is obtainable from either relation (3) or relation (4). Finally, recall that the D_i are retrieved from the lidar data by deconvolution of the lidar system response, sea state, and fluorescence lifetime as described earlier.

4. Summary and Discussion

Beam attenuation coefficient retrieval methodology by RTE inversion of airborne laser-induced and depth-resolved CDOM fluorescence is given. Two fluorescence band segments chosen within the broad CDOM fluorescence emission spectrum allow a retrieval (1) without the need for a laser beam spread model and (2) that is self-normalizing. The self-normalization arises from the fact that the fluorescence in the spectrally separated bands has a common source term. Thus the source term is eliminated during the retrieval methodology leaving the equations dependent only on the measured spectral radiances. The inversion is further enabled by the known isotropic fluorescence phase function and further simplified by the known ratio of the CDOM fluorescence distribution function for the two observational bands. Two fluorescence channels eliminate the need for a BSF model that is required when one uses the single water Raman retrieval methodology.[13]

To minimize observation of multiple scattering, the lidar is configured in a MFOV BAM. The proposed MFOV lidar receiver allows extrapolation to null or 0° FOV to optimize the BAM application. The BAM also facilitates evaluation of the fluorescence source function integral as applied to a coaxial transmitter and receiver.

Monte Carlo radiative transfer studies, and acquisition of appropriate new airborne lidar data, are planned for continued study as well as validation of this beam attenuation method.

Appendix A: Development of the Radiative Transfer Equation for Lidar-Induced Fluorescence

This theoretical development for fluorescence is similar, but not identical, to the inclusion of water Raman in the RTE.

As will be subsequently seen below, the solution of the RTE is enabled by the fact that both the CDOM fluorescence spectral redistribution function and the phase function (isotropic) are known. Development is further facilitated by the highly monochromatic character of the laser transmitter; the known directionality of the laser beam emission; the highly collimated character of the laser transmitter; the pulsed nature of the laser output that allows the depth-resolved backscattered radiances to be measured; the MFOV and known directionality of the laser receiver; for the 355 nm laser, a good choice of spectral bands within the broad CDOM fluorescence spectrum (~355–600 nm) that avoids and therefore excludes interference from the water Raman scattering at ~402 nm; pulsed laser excitation and fluorescence detection separated such that encroachment of the laser radiance into the CDOM fluorescence spectrum is easily avoided; and the receiver detectors and phototubes that are ac coupled so that dc background from solar and sky radiance is rejected.

We establish a Cartesian coordinate system with the $+z$ axis vertically downward into the ocean and the x and y axes lying in the ocean just below the atmosphere–ocean boundary. Unit vectors pointing along the positive direction of these x, y, z axes are, respectively, $\mathbf{i}, \mathbf{j}, \mathbf{k}$. In a plane-parallel medium that includes inelastic (e.g., CDOM) fluorescence and elastic scattering, the in-water time-independent RTE is[1,28-30]

$$\cos\theta \frac{\mathrm{d}L(z;\theta,\phi;\lambda)}{\mathrm{d}z} = -c(z;\lambda)L(z;\theta,\phi;\lambda) + \int_0^{2\pi}\int_0^{\pi} \beta(z;\theta',\phi'\to\theta,\phi;\lambda) \times L(z;\theta',\phi';\lambda)\sin\theta'\mathrm{d}\theta'\mathrm{d}\phi' + \int_{\lambda'}\int_0^{2\pi}\int_0^{\pi} \beta^F(z;\theta',\phi'\to\theta,\phi;\lambda'\to\lambda) \times L(z;\theta',\phi';\lambda')\sin\theta'\mathrm{d}\theta'\mathrm{d}\phi'\mathrm{d}\lambda', \quad \text{(A1)}$$

where $c(z, \lambda)$ is the beam attenuation coefficient at depth z for all absorption and scattering events at wavelength λ; $L(z; \theta, \phi; \lambda)$ is the radiance at depth z propagating and being observed in a direction specified by the polar angle θ, in water, measured with respect to the $+z$ axis, and ϕ is the azimuthal angle measured with respect to the $+x$ axis. (Primed θ' or ϕ' denote the incoming direction of source radiance at its wavelength λ'.) [Note that the required fluorescence radiance $L(z; \theta, \phi; \lambda)$ is (a) derived from the lidar temporal radiance measurement $L(t = 2zm/\nu; \theta, \phi; \lambda)$ where ν is the velocity of light, t is the two-way elapsed time, and m is the index of refraction; (b) can be contaminated with other inelastic scattering such as water Raman; (c) contains intrinsic electronic temporal decay and CDOM fluorescence lifetime decay and a sea state all of which must be deconvolved; (d) the required spatial derivative of $L(z; \theta, \phi; \lambda)$ can be calculated by using standard methods to differentiate digital data itself or to fit the digital data to an exponential(s) or a polynomial and then execute the derivative.][31] Here $\beta(z; \theta', \phi' \to \theta, \phi; \lambda)$ is the volume scattering function describing the elastic scattering of radiance at wavelength λ from direction θ', ϕ' into direction θ, ϕ. And $\beta^F(z; \theta', \phi' \to \theta, \phi; \lambda' \to \lambda)$ is the volume scattering function describing the inelastic scattering (fluorescence) of radiance at wavelength λ' from direction θ', ϕ' into radiance at wavelength λ in the direction θ, ϕ. Note that, unlike water Raman scattering, the fluorescence volume scattering function $\beta^F(\mathrm{m}^{-1}\,\mathrm{sr}^{-1})$ is depth dependent and is thus shown in Eq. (A1)

with depth dependency. Equation (A1) is valid for observational wavelengths λ and shorter wavelengths λ' where $\lambda' < \lambda$. (Superscripts or subscripts F and L denote fluorescence or the laser, respectively; when used as a subscript, L should not be confused with radiance.) The RTE of Eq. (1) does not include multiple scatter, i.e., photons that are scattered out of the transmitted beam and then again scattered back into the original transmit or receive paths. Thus this RTE will be applied to a lidar configured in a MFOV BAM to minimize observation of multiple scattering.

First, in the lowermost integral in Eq. (A1), the integration over the excitation wavelengths λ' is now addressed. Herein, the laser excitation is highly monochromatic (or spectrally very narrow) such that it is essentially a Dirac delta function at the laser wavelength λ_L' (again, note when used as a subscript, L specifically denotes the laser and is not to be confused with the radiance L). Within the λ' integration we define the directionality double integral in Eq. (A1) as

$$L^{*F}(z;\,\theta',\phi';\,\lambda') = \int_0^{2\pi}\int_0^{\pi} \beta^F(z;\,\theta',\ \phi' \to \theta,\ \phi;\,\lambda' \to \lambda) \times L(z;\,\theta',\ \phi';\,\lambda)\sin\theta' d\theta' d\phi'.$$

There is no loss of generality in extending the λ' integration in relation (3) over $\pm\infty$ such that

$$\int_{-\infty}^{+\infty} L^{*F}(z;\,\theta',\ \phi';\,\lambda')\delta(\lambda' - \lambda_L')d\lambda' = L^{*F}(z;\,\theta',\ \phi';\,\lambda_L').$$

Thus, for a single very narrow spectral linewidth excitation laser at $\lambda' = \lambda_L'$, the RTE in Eq. (A1) becomes

$$\cos\theta\,\frac{dL(z;\,\theta,\ \phi;\,\lambda)}{dz} = -c(z;\,\lambda)L(z;\,\theta,\ \phi;\,\lambda) + \int_0^{2\pi}\int_0^{\pi} \beta(z;\,\theta',\ \phi' \to \theta,\ \phi;\,\lambda)L(z;\,\theta',\ \phi';\,\lambda) \times \sin\theta' d\theta' d\phi' + \int_0^{2\pi}\int_0^{\pi} \beta^F(z;\,\theta',\ \phi' \to \theta,\ \phi;\,\lambda_L' \to \lambda) \times L(z;\,\theta',\ \phi';\,\lambda_L')\sin\theta' d\theta' d\phi'. \quad \text{(A2)}$$

Lidar observations will be made in a *specific* fluorescence band whose center wavelength is identified as $\lambda = \lambda_F$ that specifically excludes the on-wavelength or elastic wavelengths. Thus, for laser excitation at λ_L' and observation at fluorescence wavelength λ_F, the RTE becomes

$$\cos\theta\,\frac{dL(z;\,\theta,\ \phi;\,\lambda_F)}{dz} = -c(z;\,\lambda_F)L(z;\,\theta,\ \phi;\,\lambda_F) + \int_0^{2\pi}\int_0^{\pi} \beta(z;\,\theta',\ \phi' \to \theta,\ \phi;\,\lambda_F)L(z;\,\theta',\ \phi';\,\lambda_F) \times \sin\theta' d\theta' d\phi' + \int_0^{2\pi}\int_0^{\pi} \beta^F(z;\,\theta',\ \phi' \to \theta,\ \phi;\,\lambda_L' \to \lambda_F) \times L(z;\,\theta',\ \phi';\,\lambda_L')\sin\theta' d\theta' d\phi'. \quad \text{(A3)}$$

The uppermost integral is the path function for radiance $L(z;\,\theta',\phi';\,\lambda_F)$ approaching from directions θ' and ϕ' at the fluorescence wavelength λ_F that is elastically scattered into the observational directions θ and ϕ. The $L(z;\,\theta',\phi';\,\lambda_F)$ is composed of solar radiance, sky radiance, as well as fluorescence scattering radiance (resulting from the lowermost fluorescence path radiance integral to be discussed below). For example, at the fluorescence wavelength λ_F,

$$L(z;\,\theta',\ \phi';\,\lambda_F) = L_{\text{solar}}(z;\,\theta',\ \phi';\,\lambda_F) + L_{\text{sky}}(z;\,\theta',\ \phi';\,\lambda_F) + L_{\text{fluorescence}}(z;\,\theta',\ \phi';\,\lambda_F).$$

But the lidar receiver cannot observe the dc or constant signals $L_{\text{solar}}(z;\,\theta',\phi';\,\lambda_F)$ and $L_{\text{sky}}(z;\,\theta',\phi';\,\lambda_F)$ since the electronics are ac coupled and respond only to high-speed nanosecond-type pulses.[8,9] The remaining $L_{\text{fluorescence}}(z;\,\theta',\phi';\,\lambda_F)$ is pulsed radiation and can be observed by the lidar receiver. Thus Eq. (A3) becomes

$$\cos\theta\,\frac{dL(z;\,\theta,\ \phi;\,\lambda_F)}{dz} = -c(z;\,\lambda_F)L(z;\,\theta,\ \phi;\,\lambda_F) + \int_0^{2\pi}\int_0^{\pi} \beta(z;\,\theta',\ \phi' \to \theta,\ \phi;\,\lambda_F) \times L_{\text{fluorescence}}(z;\,\theta',\ \phi';\,\lambda_F)\sin\theta' d\theta' d\phi' + \int_0^{2\pi}\int_0^{\pi} \beta^F(z;\,\theta',\ \phi' \to \theta,\ \phi;\,\lambda_L' \to \lambda_F) \times L(z;\,\theta',\ \phi';\,\lambda_L')\sin\theta' d\theta' d\phi', \quad \text{(A4)}$$

wherein the uppermost double integral describes elastic scattering of the inelastic fluorescence photons (arising from the lowermost integral) into directions θ and ϕ. At fluorescence wavelengths this uppermost double-integral term now represents only laser-induced inelastic-to-elastic or second-order scattering. But the lidar is in a coaxial monostatic mode and the important elastic backscattering portion of $\beta(z;\,\theta',\phi' \to \theta,\phi;\,\lambda_R)$ is effectively $\beta(\pi)$ whose value in coastal waters is nominally $\sim 10^{-3}\ \text{m}^{-1}\ \text{sr}^{-1}$ at 514 nm.[1] Thus the unscattered isotropic fluorescence at the instant of emission from the fluorophore (pro-

duced and described by the lowermost integral) that is isotropically propagating vertically toward the lidar receiver is $\sim 10^3$ stronger than the elastically backscattered fluorescence component. In fact, these inelastic fluorescence photons that are elastically forward scattered upward toward the lidar receiver are strongly peaked and can actually be $\sim 10^6$ stronger than the elastically backscattered fluorescence since the coastal water elastic volume scattering function in the forward direction is[1] $\sim 10^{-3}\ \mathrm{m}^{-1}\ \mathrm{sr}^{-1}$ (i.e., the ratio of the elastically forward-scattered fluorescence photons to the elastically backscattered fluorescence photons is $\sim(10^{+3}\ \mathrm{m}^{-1}\ \mathrm{sr}^{-1})/(10^{-3}\ \mathrm{m}^{-1}\ \mathrm{sr}^{-1}) = 10^6$. These arguments are in agreement with others.[32,33] Recall also that small-angle forward scatter is essentially equivalent to no scatter at all; and this is especially important for our lidar in the BAM whose FOV can be extrapolated to 0°. [At the risk of being redundant, a slightly more specific example for CDOM fluorescence is as follows. At the instant CDOM fluorescence leaves the molecular fluorophore in isotropic propagation, the spherical portion in the deeper and the lowermost hemisphere has a small amount returned toward the lidar receiver via elastic backscattering (and at a wavelength of 514 nm, the volume scattering coefficient in the backward direction in coastal water is[1] $\sim 1.03 \times 10^{-3}\ \mathrm{m}^{-1}\ \mathrm{sr}^{-1}$)]. Concurrently, the spherically propagating fluorescence in the uppermost hemisphere is forward scattered upward toward the lidar receiver by particulates typically having highly peaked elastic phase functions, and at 514 nm the volume scattering coefficient in the forward direction is[1] $\sim 6.5 \times 10^2\ \mathrm{m}^{-1}\ \mathrm{sr}^{-1}$. Thus the ratio of the elastically forward-scattered CDOM fluorescence propagating toward the lidar receiver to the backscattered portion propagating upward from the lower hemisphere is $\sim(6.5 \times 10^2\ \mathrm{m}^{-1}\ \mathrm{sr}^{-1})/(10^{-3}\ \mathrm{m}^{-1}\ \mathrm{sr}^{-1}) = 6.5 \times 10^5$. Again, the highly peaked forward scatter toward the lidar receiver within the small angle of the lidar FOV is essentially equivalent to no scatter at all and is described by the $-cL$ term of the RTE. Thus again this second-order elastically scattered fluorescence is ignored compared with the inelastic term. If the CDOM concentration is rather high, then the absorption is correspondingly high and multiple elastic scattering becomes even less important. In short, the fluorescence becomes the sole light source term for the eventual measurement of the beam attenuation coefficient.

Accordingly, the uppermost integral is very small and ignorable compared with the lowermost fluorescence term and yields

$$\cos\theta\,\frac{\mathrm{d}L(z;\,\theta,\,\phi;\,\lambda_F)}{\mathrm{d}z} = -c(z;\,\lambda_F)L(z;\,\theta,\,\phi;\,\lambda_F) + \int_0^{2\pi}\int_0^{\pi}\beta^R(\theta',\,\phi'\to\theta,\,\phi;\,\lambda_L'\to\lambda_F) \times L(z;\,\theta',\,\phi';\,\lambda_L')\sin\theta'\,\mathrm{d}\theta'\mathrm{d}\phi'. \quad \text{(A5)}$$

The fluorescence source term, the lowermost term of Eq. (A5), will now be evaluated. The fluorescence volume scattering function can be multiplicatively partitioned[1]:

$$\beta^F(z;\,\theta',\,\phi'\to\theta,\,\phi;\,\lambda_L'\to\lambda_F) = a^F(z,\,\lambda_L')f^F(\lambda_L'\to\lambda_F)\tilde{\beta}^F(\theta',\,\phi'\to\theta,\,\phi), \quad \text{(A6)}$$

where $a^F(z,\lambda_L')$, $f^F(\lambda_L'\to\lambda_R)\tilde{\beta}^F(\theta',\phi'\to\theta,\phi)$ are, respectively, the fluorescence spectral absorption coefficient (in units of m^{-1}), wavelength redistribution function,[1] and phase function. Partitioned in this way, only the phase function remains within the directionality integral, and the inelastic fluorescence source term becomes

$$L^{*F}(z;\,\theta,\,\phi;\,\lambda_L') = a^F(z,\,\lambda_L')f^F(\lambda_L'\to\lambda_F) \times\int_0^{2\pi}\int_0^{\pi}\tilde{\beta}^F(\theta',\,\phi'\to\theta,\,\phi) \times L(z;\,\theta',\,\phi';\,\lambda_L')\sin\theta'\,\mathrm{d}\theta'\mathrm{d}\phi'. \quad \text{(A7)}$$

Note that the fluorescence absorption coefficient $a^F(z,\lambda_L')$ is unknown and is a single spectral value at any depth. As a result of the absorption at the highly monochromatic laser excitation λ_L', only one fluorescence scattering redistribution spectrum, $f^F(\lambda_L'\to\lambda_F)$, is generated[1] (see Fig. 5.7 in Mobley[1]). The lidar receiver narrowband fluorescence filter has a bandpass $\Delta\lambda$ centered at λ_{Y1} [and produces a single spectrally integrated value labeled $f^F(\lambda_L'\to\lambda_{Y1})$] within the broader fluorescence redistribution function $f^F(\lambda_L'\to\lambda_F)$. The directionality terms $\tilde{\beta}^F(\theta',\phi'\to\theta,\phi)$ and $L(z;\theta',\phi';\lambda_L')$ are dependent on the lidar geometry, and further simplifications to the fluorescence source term are possible since the lidar is in the BAM. With the BAM viewing geometry, the included angle between transmitted and received photons is $\mathbf{k}\cdot-\mathbf{k} = -1$ or $\theta = \pi = 180°$ and $\cos\theta = -1$. Of course the fluorescence phase function is known: $\tilde{\beta}^F(\theta',\phi'\Rightarrow\theta,\phi) = \tilde{\beta}^F(\psi) = 1/4\pi(\mathrm{sr}^{-1})$ (see Chap. 5 of Mobley[1]) so that Eq. (A5) becomes

$$-\frac{\mathrm{d}L(z;\,\theta,\,\phi;\,\lambda_F)}{\mathrm{d}z} = -c(z;\,\lambda_F)L(z;\,\theta,\,\phi;\,\lambda_F) + a^F(z,\,\lambda_L')f^F(\lambda_L'\to\lambda_F) \times\tilde{\beta}^F(\psi)\int_0^{2\pi}\int_0^{\pi}L(z;\,\theta',\,\phi';\,\lambda_L') \times\sin\theta'\,\mathrm{d}\theta'\mathrm{d}\phi'. \quad \text{(A8)}$$

Relation (A8) is used within the main text of this paper as the starting point for the retrieval of the beam attenuation coefficient from CDOM fluorescence.

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References and Notes

1. C. D. Mobley, *Light and Water, Radiative Transfer in Natural Waters* (Academic, 1994), Chap. 5. The errata can be found at ftp.sequoiasci.com within the file/pub/outgoing/LightandWaterErrata.pdf. Note: at the time of publication, Mobley's *Light and Water* is now available on CD, with the errors corrected. The CD is available from Mobley, or the contents of the CD can be downloaded from http://www.curtismobley.com/curts_work.htm or from http://www.curtismobley.com/LightandWater.zip.
2. J. K. B. Bishop, "Transmissometer measurement of POC," Deep-Sea Res. I **46,** 353–369 (1999).
3. J. K. B. Bishop, S. E. Calvert, and M. Y. S. Moon, "Spatial variability of POC in the northeast Subarctic Pacific," Deep-Sea Res. II **46,** 2699–2733 (1999).
4. F. E. Hoge, C. W. Wright, P. E. Lyon, R. N. Swift, and J. K. Yungel, "Satellite retrieval of inherent optical properties by inversion of an oceanic radiance model: a preliminary algorithm," Appl. Opt. **38,** 495–504 (1999).
5. F. E. Hoge, C. W. Wright, P. E. Lyon, R. N. Swift, and J. K. Yungel, "Satellite retrieval of the absorption coefficient of phytoplankton phycoerythrin pigment: theory and feasibility status," Appl. Opt. **38,** 7431–7441 (1999).
6. F. E. Hoge, A. Vodacek, R. N. Swift, J. K. Yungel, and N. V. Blough, "Inherent optical properties of the ocean: retrieval of the absorption coefficient of chromophoric dissolved organic matter from airborne laser spectral fluorescence measurements," Appl. Opt. **34,** 7032–7038 (1995).
7. F. E. Hoge, M. E. Williams, R. N. Swift, J. K. Yungel, and A. Vodacek, "Satellite retrieval of the absorption coefficient of chromophoric dissolved organic matter in continental margins," J. Geophys. Res. **100,** 24847–24854 (1995).
8. F. E. Hoge, R. E. Berry, and R. N. Swift, "Active-passive airborne ocean color measurement. 1: Instrumentation," Appl. Opt. **25,** 39–47 (1986).
9. F. E. Hoge, R. N. Swift, and J. K. Yungel, "Active-passive airborne ocean color measurement. 2: Applications," Appl. Opt. **25,** 48–57 (1986).
10. F. E. Hoge, A. Vodacek, and N. V. Blough, "Inherent optical properties of the ocean: retrieval of the absorption coefficient of chromophoric dissolved organic matter from fluorescence measurements," Limnol. Oceanogr. **38,** 1394–1402 (1993).
11. R. J. Exton, W. M. Houghton, W. Esaias, R. C. Harriss, F. H. Farmer, and H. H. White, "Laboratory analysis of techniques for remote sensing of estuarine parameters using laser excitation," Appl. Opt. **22,** 54–64 (1983).
12. F. E. Hoge, C. W. Wright, P. E. Lyon, R. N. Swift, and J. K. Yungel, "Inherent optical properties imagery of the western North Atlantic Ocean: horizontal spatial variability of the upper mixed layer," J. Geophys. Res. **106,** 31129–31140 (2001).
13. F. E. Hoge is preparing a manuscript with the title "Oceanic beam attenuation coefficient retrieval by radiative transfer equation inversion of airborne water Raman backscattering. 1: Theory."
14. C. D. Clark, J. Jimenez-Morais, G. Jones, E. Zanardi-Lamardo, C. A. Moore, and R. G. Zika, "A time-resolved fluorescence study of dissolved organic matter in a riverine to marine transition zone," Mar. Chem. **78,** 121–135 (2002).
15. A. Bricaud, A. Morel, and L. Prieur, "Absorption by dissolved organic matter of the sea (yellow substance) in the UV and visible domains," Limnol. Oceanogr. **26,** 43–53 (1981).
16. F. E. Hoge, R. N. Swift, J. K. Yungel, and A. Vodacek, "Fluorescence of dissolved organic matter: a comparison of North Pacific and North Atlantic Oceans during April 1991," J. Geophys. Res. **98,** 22779–22787 (1993).
17. C. W. Wright, F. E. Hoge, R. N. Swift, J. K. Yungel, and C. R. Schirtzinger, "Next-generation NASA airborne oceanographic lidar system," Appl. Opt. **40,** 336–342 (2001).
18. F. E. Hoge and R. N. Swift, "Airborne detection of oceanic turbidity cell structure using depth-resolved laser-induced water Raman backscatter," Appl. Opt. **22,** 3778–3786 (1983).
19. F. E. Hoge and R. N. Swift, "Experimental feasibility of the airborne measurement of absolute oil fluorescence spectral conversion efficiency," Appl. Opt. **22,** 37–47 (1983).
20. H. R. Gordon, "Interpretation of airborne oceanic lidar: effects of multiple scattering," Appl. Opt **21,** 2996–3001 (1982).
21. G. Roy, L. R. Bissonnette, C. Bastille, and G. Vallee, "Retrieval of droplet-size density distribution from multiple-field-of-view cross-polarized lidar signals: theory and experimental validation," Appl. Opt. **38,** 5202–5211 (1999).
22. L. Bissonnette, G. Roy, L. Poutier, S. G. Cober, and G. A. Isaac, "Multiple-scattering lidar retrieval method: tests on Monte Carlo simulations and comparisons with *in situ* measurements," Appl. Opt. **41,** 6307–6324 (2002).
23. L. R. Bisonnette and D. L. Hunt, "Multiple scattering lidar," Appl. Opt. **29,** 5045–5046 (1990).
24. D. L. Hutt, L. R. Bissonnette, and L. Durand, "Multiple field of view lidar returns from atmospheric aerosols," Appl. Opt. **33,** 2338–2348 (1994).
25. F. E. Hoge, "Validation of satellite-retrieved oceanic inherent optical properties: proposed two-color elastic backscatter lidar and retrieval theory," Appl. Opt. **42,** 7191–7196 (2003).
26. K. J. Voss, "A spectral model of the beam attenuation coefficient in the ocean and coastal areas," Limnol. Oceanogr. **37,** 501–509 (1992).
27. A. H. Bernard, W. S. Pegau, and J. R. V. Zaneveld, "Global relationships of the inherent optical properties of the oceans," J. Geophys. Res **103,** 24955–24968 (1998).
28. H. R. Gordon, "Contribution of Raman scattering to water-leaving radiance: a reexamination," Appl. Opt. **38,** 3166–3174 (1999).
29. Y. Ge, H. R. Gordon, and K. J. Voss, "Simulation of inelastic-scattering contributions to the irradiance field in the ocean: variation in Fraunhofer line depths," Appl. Opt. **32,** 4028–4036 (1993).
30. G. C. Boynton and H. R. Gordon, "Irradiance inversion algorithm for estimating the absorption and backscattering coefficients of natural waters: Raman-scattering effects," Appl. Opt. **39,** 3012–3022 (2000).
31. W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipies in Fortran,* 2nd ed. (Cambridge U. Press, 1992).
32. A. V. Malinka and E. P. Zege, "Analytical modeling of Raman lidar return, including multiple scattering," Appl. Opt. **42,** 1075–1081 (2003).
33. I. Katzev, E. Zege, A. Prikhach, and I. Polansky, "Efficient technique to determine backscattered light power for various atmospheric and ocean soundings and imaging systems," J. Opt. Soc. Am. A **14,** 1338–1346 (1997).