Advection of magnetic energy as a source of power for auroral arcs

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[1] We examine the energetics of a system wherein large-scale magnetospheric convection causes plasma to flow across two-dimensional sheets of field-aligned current (FAC). This scenario requires that the FACs be held stationary or move slowly relative to the background flow, for example through their connection to conductivity enhancements in the lower ionosphere. The key result is that plasma convection across quasi-static current sheets at speed V_d implies cross-field transport of magnetic energy at a rate proportional to $V_d \delta B^2/2\mu_0$, where δB is the magnetic perturbation associated with the FAC. Poynting's theorem shows that this energy is available to enhance the field-aligned component of Poynting flux, to accelerate particles via electric fields, or both. We show that, for nominal values of field-aligned current ($\sim$10 μA/m^2) and cross-current-sheet convection ($\sim$100 m/s), the net field-aligned energy flux made available through cross-field advection can contribute of the order of 1 mW/m^2 to auroral energy fluxes (electromagnetic plus particle), and conceivably can dominate the energy budget in more intense arcs. While our analysis is applied to an idealized, quasi-static, two-dimensional system, the mechanism it illustrates could play an important role in more dynamic and highly-structured auroral forms as well. **Citation:** Knudsen, D. J., J. K. Burchill, E. F. Donovan, and V. M. Uritsky (2011), Advection of magnetic energy as a source of power for auroral arcs, *Geophys. Res. Lett.*, *38*, L24103, doi:10.1029/2011GL049661.

1. Introduction

[2] Auroral arcs are longitudinally elongated regions of enhanced energy flux into the upper atmosphere in the form of accelerated electron beams and Joule heating in the lower ionosphere. Arc models commonly involve a distant source that applies electric fields (voltage generator) or field-aligned currents (current generator) on magnetic field lines that thread arcs [e.g., *Lysak*, 1985]. Significant challenges stand in the way of identifying a source that can sustain observed levels of energy dissipation within stable arcs, and that can also explain their basic morphology: long, thin, often comprising multiple parallel structures, and often lasting tens of minutes with little variation [e.g., *Lessard et al.*, 2007]. Furthermore, arcs are found over a very wide range of magnetic latitudes corresponding to regions ranging from the inner magnetosphere to the plasma sheet boundary layer and even the polar cap, posing a serious problem for any theory that is tied to a specific region or boundary, such as a pressure gradient at the inner edge of the plasma sheet, or a mid-tail reconnection line, for example. The existence of an upward field-aligned electrical current alone is not sufficient to generate an auroral arc. In addition there must be a mechanism to accelerate current carriers, and to account for downward fluxes of electromagnetic energy at levels comparable to those carried by electrons [*Evans et al.*, 1977; *Mallinckrodt and Carlson*, 1985].

[3] This paper explores a mechanism intended to address these challenges. With this mechanism, structured enhancements of auroral energy flux (auroral arcs) result from magnetospheric convection and consequent transport of magnetic energy across field-aligned current (FAC) systems such as the Region 1/2 current systems, which can be wider than the arcs embedded within them. The concept of magnetic energy as a source of power for auroral arcs has been discussed by previous authors [e.g., *Haerendel et al.*, 1993] in the context of shrinking and expanding current circuits. Our study shows how magnetic energy transport can persist indefinitely in a quasi-static steady state and contribute significantly to observed levels of dissipation.

[4] The superposition of field-aligned currents and plasma convection was studied initially by *Maltsev et al.* [1977] and *Mallinckrodt and Carlson* [1978]. They showed that an assumed source of FAC fixed relative to plasma convection, added to field-aligned propagation of the resulting disturbance at the Alfvén speed V_A, leads to electromagnetic structures that are stationary in the source frame. These disturbances, known as stationary Alfvén waves, comprise surfaces of constant field and field-aligned current that are tilted slightly in the direction of background convection with respect to **B** at an angle $\alpha = \tan^{-1}(V_d/V_A)$, where α is typically of the order of 10^{-4} radians. Whereas those initial studies assumed thin current sheets, *Knudsen* [1996] applied a two-fluid model to the interior of finite-width, drifting FACs to show they are intrinsically susceptible to structuring in density and electron energy at a scale that does not depend on the structure of an assumed source and that is not imposed by the ionosphere. Specifically, an initially unstructured FAC can structure into sheets of accelerated electron beams and density depletions under the action of cross-FAC plasma drift. The present study addresses the energetics of such a system.

[5] There have been a few experimental attempts to measure plasma flow across arcs and, by extension, across the current sheets in which they are embedded. *Haerendel et al.* [1993] and *Frey et al.* [1996] used the EISCAT radar and ground-based cameras to infer cross-arc flows of the order of 100–200 m/s. *Robinson et al.* [1981] reported a constant electric field of 7 mV/m tangential to an auroral arc as measured by a sounding rocket that was approximately constant across the arc; *de la Beaujardiere et al.* [1981] reported a similar observation using satellite and radar data. Motivated by these observations, in the following section we derive an expression for the effect of cross-

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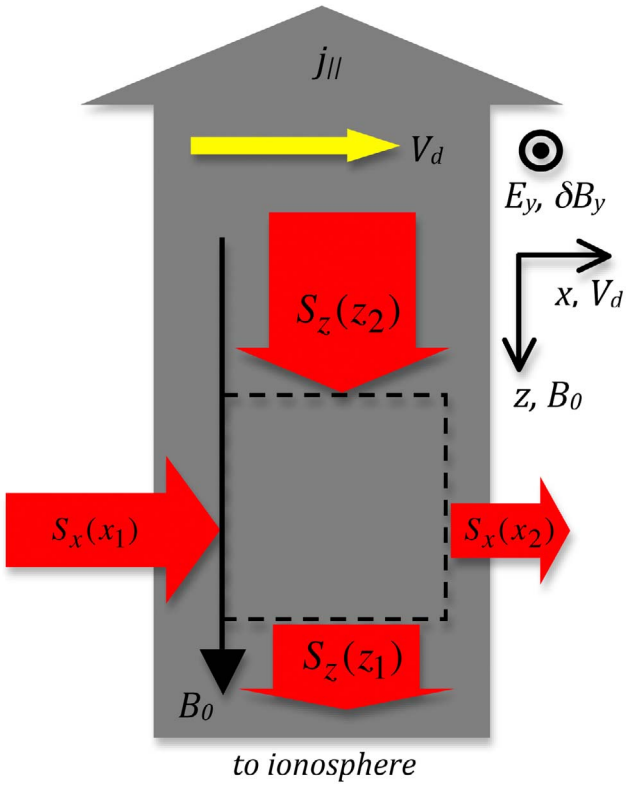

Figure 1. Cross section of a two-dimensional field-aligned current sheet, viewing nominally eastward (e.g., in the evening-side auroral zone) for a westward background electric field E_y.

current sheet flows on the energy budget of auroral flux tubes.

2. Derivation

[6] We consider a section of a quasi-static, two-dimensional magnetic-field-aligned current sheet extended in the y direction (nominally westward, or dawn-to-dusk in the nightside auroral zone) as shown in Figure 1. The reason for this choice is to isolate and illustrate a source of energy that can operate in a quasi-static steady state, consistent with observations of evening-side arcs prior to substorm breakup, for example. Assuming no time variation allows us to isolate a specific mechanism that we believe has been overlooked in most previous models of quasi-static arcs, however we believe that this mechanism could play a role in more structured and dynamic arcs as well.

[7] For the calculation below we assume that the magnetic field $\mathbf{B}$ is dominated by a strong background field $B_0\hat{z}$, as shown in Figure 2. Field-aligned current (parallel to $\mathbf{B}$) in this geometry generates a predominantly y-directed magnetic perturbation that varies in x: $\partial B_y/\partial x = \mu_0 j_z$. In quasi-static two-dimensional sheets the conditions $\nabla \times \mathbf{E} = 0$ and $\partial/\partial y = 0$ dictate that E_y must be uniform over the dimensions of the arc ($\partial E_y/\partial x = 0$), implying a source far from the arc system. Such a large-scale field is consistent for example with the dawn-dusk electric field that originates from the solar wind-magnetosphere interaction and permeates much of the magnetosphere, manifesting as earthward plasma convection in the tail. In the following, we assume that the current sheet is stationary relative to the large-scale convective flow, for example via connection at its lower boundary to a conductivity enhancement in the ionosphere. The resulting cross-sheet plasma flow is assumed to be uniform along the length (in y) of the current sheet. Electric field components E_x and E_z can vary with x and z in general, though must remain curl-free under the quasi-static approximation.

[8] The integral form of Poynting's theorem states

$$\oint \mathbf{S} \cdot d\mathbf{A} = -\int \mathbf{J} \cdot \mathbf{E} dV - \frac{\epsilon_0}{2}\frac{\partial}{\partial t}\int \left(c^2B^2 + E^2\right) dV \qquad (1)$$

where the Poynting vector $\mathbf{S} \equiv \mathbf{E} \times \mathbf{B}/\mu_0$ and the other definitions are standard. The first integral on the right represents the rate of exchange of electromagnetic and

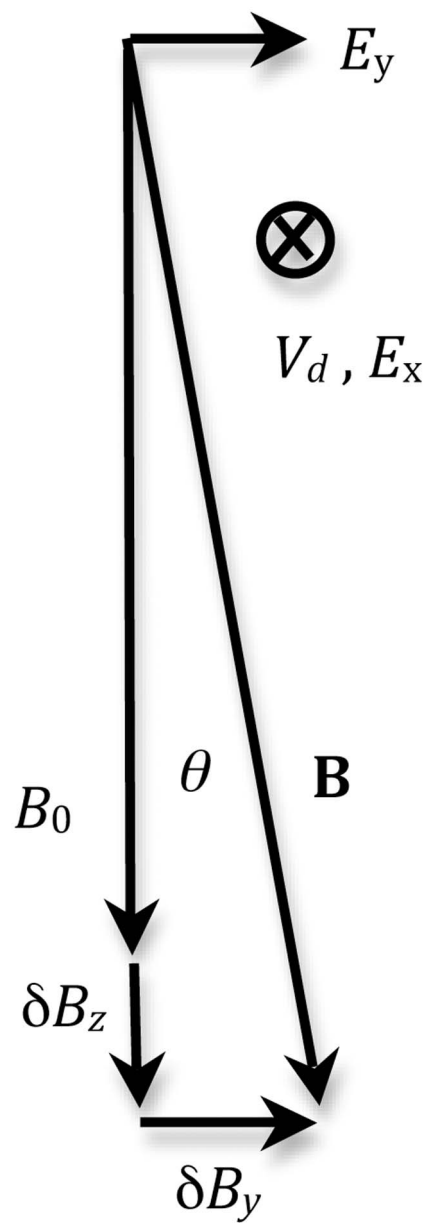

Figure 2. Magnetic and electric field components viewing nominally southward (e.g., in the evening side-auroral zone) for a westward background electric field E_y.

mechanical energy. The second is the time rate of change of total electromagnetic energy, equal to zero in the quasi-static case. The integral on the left will be carried out over the surface of the rectangular prism shown in cross section as a dashed rectangle in Figure 1, having dimensions Δx, Δy, Δz.

2.1. Surfaces of Constant *y*

[9] The value of $S_y = \mathbf{S} \cdot \hat{y}$ calculated on surfaces of constant y is non-zero in general, however it does not change with y in a 2-D geometry and therefore makes no net contribution to the overall Poynting flux budget, and can be ignored.

2.2. Top and Bottom Surfaces

[10] The top and bottom surfaces are at $z = z_2$ and $z = z_1$ respectively. In general the normal component of the Poynting vector at these surfaces is

$$S_z = \left(E_x B_y - E_y B_x\right)/\mu_0 \tag{2}$$

The term $E_x B_y/\mu_0$ usually dominates the Poynting flux budget and most studies consider only this contribution. The second term, proportional to B_x, can result from a y-directed, arc aligned current, which can be significant in the Hall and Pedersen current layers in the lower ionosphere, and can affect the vertical component of Poynting flux [*Haerendel*, 2008; *Richmond*, 2010]. For the purposes of this study we place the lower boundary of the integration surface far enough above the ionosphere that B_x can be neglected.

2.3. Surfaces of Constant *x*

[11] The new result of this paper concerns the remaining component of Poynting flux,

$$S_x = \left(E_y B_z - E_z \delta B_y\right)/\mu_0. \tag{3}$$

In the absence of any FAC, S_x takes on a constant value of $E_y B_0$ and makes no net contribution to the left side of equation (1). Under the approximations described above, FACs generate a magnetic field perturbation $\delta \boldsymbol{B} = \delta B_y \hat{y} + \delta B_z \hat{z}$ to produce a total field $\mathbf{B} = (B_0 + \delta B_z)\hat{z} + \delta B_y \hat{y}$.

[12] We can write δB_z in terms of δB_y by noting that $\partial \delta B_z/\partial x = -\mu_0 j_y$, and that far from the collisional currents in the ionosphere, $j_y = j_\parallel \sin\theta \approx j_\parallel \delta B_y/B_0$ (accurate to first order in the quantity $\delta B_y/B_0$), so that

$$\frac{\partial \delta B_z}{\partial x} = -\mu_0 j_\parallel \frac{\delta B_y}{B_0} = -\frac{1}{2B_0}\frac{\partial \delta B_y^2}{\partial x} \tag{4}$$

Integrating once,

$$\frac{\delta B_z}{B_0} = -\frac{\delta B_y^2}{2B_0^2} \tag{5}$$

The z-directed electric field in (3) requires special attention. Typically, E_z is equated with $E_\parallel$, the field component parallel to **B**. However this is not appropriate in the presence of the large-scale convection field E_y. Instead, **E** must be projected into the direction of the total **B** including any contributions from field-aligned currents:

$$E_\parallel \equiv \mathbf{E} \cdot \hat{\mathbf{b}} = \frac{\delta B_y E_y + (B_0 + \delta B_z) E_z}{B} \tag{6}$$

[13] This reduces to

$$E_\parallel = \frac{\delta B_y}{B_0} E_y + E_z + O\left(\delta B_y^4/B_0^4\right) \tag{7}$$

This distinction between $E_\parallel$ and E_z was pointed out by *Mallinckrodt and Carlson* [1978], who argued that field-aligned current perturbations propagate as Alfvén waves that change both δB_y and E_z in a manner that maintains $E_\parallel = 0$. *Seyler* [1990] showed that $E_\parallel$ cannot be assumed to be zero in general due to finite electron inertia. *Knudsen* [1996] showed that $E_\parallel$ resulting from electron inertia can be important even in quasi-static situations, namely in the interior of field-aligned current sheets that have a normal component of plasma flow.

[14] Substituting (5) and (7) into (3) and dropping the constant term $E_y B_0$,

$$S_x = V_d \frac{\delta B_y^2}{2\mu_0} - E_\parallel \frac{\delta B_y}{\mu_0} \tag{8}$$

where $V_d = E_y/B_0$. The first term on the right-hand side provides an interpretation of cross-field Poynting flux S_x: it represents advection of magnetic energy associated with the perturbation magnetic field. In the presence of field-aligned current, δB_y varies in the direction of the background convection (x), leading to a net inflow (or outflow) of magnetic energy onto (or from) individual flux tubes. Interestingly, in this situation magnetic energy is transported along with convection even though magnetic field δB_y is not.

[15] The net power injected onto the flux tube over a small surface is $P_\perp = \Delta y \Delta z (S_{x,1} - S_{x,2})$. This can be written as power available per unit transverse area of flux tube as:

$$F_\parallel \equiv \frac{P_\perp}{\Delta x \Delta y} = \Delta z \left(\frac{S_{x,1} - S_{x,2}}{\Delta x}\right) \tag{9}$$

[16] Taking the limit of small Δx allows us to approximate (9) in terms of Ampere's law ($\partial \delta B_y/\partial x = \mu_0 j_\parallel$, with $j_z \approx j_\parallel$):

$$F_\parallel \approx -\Delta z \frac{E_y}{B_0} \delta B_y j_\parallel + \Delta z \frac{\partial}{\partial x}\left(E_\parallel \delta B_y/\mu_0\right) \tag{10}$$

[17] The minus sign signifies that magnetic energy $\delta B_y^2/2\mu_0$ increasing in the direction of convection implies that energy is being extracted from convecting flux tubes, and vice versa. For the case of constant $E_\parallel$ the term on the right is proportional to $E_\parallel j_\parallel$, which represents conversion of electrical to mechanical energy in the form of (predominantly) electron acceleration parallel to **B**.

[18] In order to estimate the significance of magnetic energy advection alone we consider the case $E_\parallel = 0$ and integrate equation (10) over a section of a side of the flux tube in the y-z plane. At this point we relax the assumption of rectangular geometry and account for field-line mapping, starting with values at the base of the field line: $E_{y,0}$, $B_{0,0}$, $j_{\parallel,0}$, $B_{y,0}$. Mapping factors for B_0 and $j_\parallel$ cancel since $B_0 \propto j_\parallel$. We assume that E_y is proportional to $d_y^{-1}(z)$, where d_y is the longitudinal (y) separation of field lines at constant latitude in a flux tube (see Figure 3), normalized to their separation at the base of the flux tube. This approximation is justified by the requirement that $\oint \vec{E} \cdot \vec{dl} = 0$, taking a rectangular

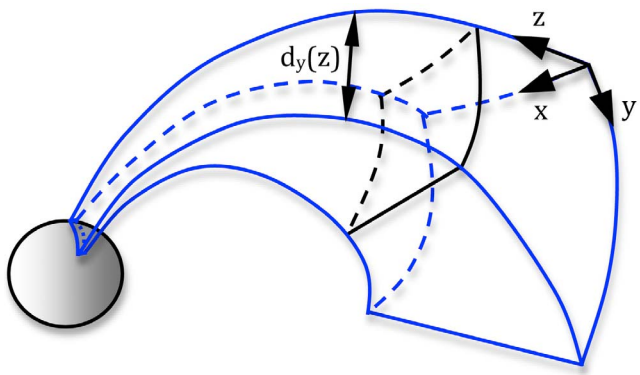

Figure 3. Integration contour (in black) used to determine the dependence of δB_y and E_y on field line separation $d_y(z)$.

contour inside the 2-D current sheet lying in the x-y plane at fixed z and by noting that the magnetic field lines that thread the four corners of the contour are equipotentials if $E_\parallel = 0$.

[19] Similarly, $\oint B \cdot dl$ over the same contour (Figure 3) is proportional to the total current I carried by the flux tube, and therefore $\delta B_y \propto d_y^{-1}(z)$ if I remains constant, since the integrations in the $\pm x$ directions cancel. Finally, taking into account the fact that integration over longitude must be carried out over a variable y' that accounts for field-line spreading through the relation $\partial y' = (d_y(z)/\Delta y_0)\partial y$:

$$F_\parallel \approx -\frac{j_{\parallel,0}}{B_{0,0}} \int_{z_1'}^{z_2'} \int_{y_1}^{y_2} \frac{E_{y,0}}{d_y(z)} \frac{\delta B_{y,0}}{d_y(z)} \frac{d_y(z)}{\Delta y_0} \partial y \partial z' \qquad (11)$$

where the zero subscript indicates values at the foot of the flux tube. Use of the integration variable z' (rather than z) represents a subtlety that arises from the fact discussed in the Introduction that plasma convection in the x direction leads to stationary wavefronts tilted slightly with respect to **B**, and the surface of integration should be aligned with these surfaces. This has little practical effect on the integration but is important conceptually.

[20] We numerically integrated the net factor d_y^{-1} along field lines from just above the ionosphere to the equatorial plane using the *Tsyganenko* [1989] model with $K_p = 1$ to compute normalized longitudinal field line separation. The result is a nearly constant value of 1.5–1.6R_E for L values ranging from 6.5 to 9.0, for example. In other words, the effective "capture area" for cross-field Poynting flux between the ionosphere and equatorial plane is $\sim 1.5\Delta y_0 R_E$. More than 75% of the integral is due to the first $4R_E$ of field-line length above the ionosphere. The additional energy flux (field and particle) due to magnetic energy advection and mapped to the ionosphere is

$$F_\parallel \approx -1.5 R_E \frac{E_{y,0}}{B_{0,0}} j_{\parallel,0} \delta B_{y,0} \qquad (12)$$

[21] Using values characteristic of a moderately intense, stable arc in the ionosphere: $E_{y,0}$ = 5 mV/m (westward), corresponding to southward convection of ~100 m/s normal to the current sheet in a background field of B_0 = 50,000 nT; $B_{y,0}$ = 100 nT (also westward); and $j_{\parallel,0} = -10$ μA/m^2 (upward FAC). This leads to an enhanced field-aligned energy flux (particle plus Poynting flux) from cross-field Poynting flux of 1 mW/m^2 when mapped to the ionosphere. This value, while modest, is nevertheless 40 times larger than the height-integrated Joule dissipation resulting from the zonal field E_y alone driving current through a height-integrated ionospheric Pedersen conductivity Σ_P = 1 S, for example. Comparing with the dominant component of the downward-directed Poynting flux $S_z = E_x B_y/\mu_0 = \Sigma_P E_x^2$, a nominal value of E_x = 100 mV/m (consistent with the above estimates through the relation $\mu_0 E_x/B_y = \Sigma_P^{-1}$ [*Kelley et al.*, 1991]) gives a dissipation of 10 mW/m^2, ten times larger than the advective effect represented by equation (12). However, the relative importance of advective transport is proportional to $j_\parallel$, and observations have shown structured currents can reach tens or even hundreds of μA/m^2 [*St. Maurice et al.*, 1996, and references therein], although the most intense currents tend to be downward. In summary, cross-field Poynting flux can account for ~10% of the total energy flux in an arc with $j_\parallel \sim -10\mu$A/m^2, but could dominate the power budget of flux tubes within arcs that are more intense or more rapidly drifting.

3. Discussion

[22] The calculation in the previous section demonstrates the need to modify the traditional view of a magnetospheric generator of auroral arcs that is magnetically conjugate to the arc itself. In that view, energy from the generator is coupled to the ionosphere along auroral field lines; auroral arcs are interpreted to be signatures of upward current resulting from the interaction between the distant generator and ionosphere. Their elongated structure is interpreted as a map of the assumed generator itself, in some cases modified by additional effects, for example Alfvén wave interference [e.g., *Haerendel et al.*, 1993; *Lysak*, 1985]. Our study uses instead as its starting point a quasi-static field-aligned current system having a spatial scale that can be much larger than the arc itself, along with a large-scale, unstructured dawn-to-dusk electric field. While the rate of magnetic energy advection we find, $V_d \delta B_y^2/2\mu_0$, is the same as would occur if regions of enhanced magnetic field were to be transported into regions of lower field, in our case the magnetic field configuration does not change. An earlier study [*Knudsen*, 1996] showed that the same system we study here is susceptible to spatially periodic internal structuring leading to inverted-V-shaped sheets of electron acceleration on scales transverse to **B** of one to hundreds of electron inertial lengths c/ω_{pe}, comparable to observed scales of auroral arcs.

[23] While the ionosphere is not included explicitly in our model, its effects are implied in two ways. First, we assume the existence of a mechanism to hold or anchor the FAC steady against the large-scale background convection. Magnetic field-line tying in the conductive ionosphere is a plausible mechanism for this. Second, the ionosphere moderates field-aligned currents, and therefore the amount of energy available from magnetic advection. For example, a non-conducting ionosphere would lead to $j_\parallel = 0$ in equation (12).

[24] While not yet included explicitly in our model, the interaction of large-scale plasma convection with ionospheric conductivity enhancements has been considered in the stationary Alfvén wave theories of *Maltsev et al.* [1977] and

Mallinckrodt and Carlson [1978], and studied more extensively in the so-called ionospheric feedback theory of auroral arcs [e.g., *Atkinson*, 1970; *Sato*, 1978; *Hasegawa et al.*, 2010]. In ionospheric feedback models, plasma convection across ionospheric conductivity enhancements stimulates field-aligned currents that in turn can reinforce the conductivity through electron precipitation, leading to quasi-static arc-like structures. The energy flux in these feedback arcs is drawn from Poynting flux associated with the background plasma convection [*Lysak and Song*, 2002], which is typically of order 0.1 mW/m^2 ($|E| \sim 10$ mV/m; $\Sigma_P \sim 1$ S), whereas arcs tend to dissipate energy at rates tens or hundreds of times higher than this. In order to obtain enhanced energy fluxes similar to those observed within arcs, *Lysak and Song* [2002] superimposed an additional generator structured on the scale of the arc itself. Our study and that of *Knudsen* [1996] are complementary to feedback models in that they draw on magnetic energy stored within and transported across FACs.

[25] Another key aspect of this study that distinguishes it from most treatments of magnetosphere-ionosphere coupling is that we assume that the large-scale cross-arc convection is independent of, and is not affected by, the relatively smaller-scale FAC systems and arc-associated electric fields embedded within them. As discussed in Section 2, this is justified by Faraday's law applied to the two-dimensional geometry of quasi-static arcs. In terms of Alfvén wave theory, arc-related fields and currents correspond to the shear Alfvén mode (even at zero frequency f), whereas the large-scale background field represents compressional or fast-mode fields. At low frequencies, compressional-wave fields are evanescent except when their cross-**B** wavelengths are larger than V_A/f. In this paper we have restricted our attention to FACs far above the ionosphere where these two modes are uncoupled. In the ionosphere they are coupled by Hall currents. For example, E_y interacting with a narrow conductivity channel will lead to divergent Hall currents in the cross-arc direction that must close via FACs. While the amount of magnetic energy advection we have calculated does not depend explicitly on this effect, the addition of ionospheric effects to our model of plasma drift through FAC sheets is an important topic for future research.

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