X-ray and Gamma-ray focusing and interferometry

Gerald K. Skinner[a,b]

[a] CRESST & NASA-GSFC, Greenbelt. MD 20771, USA;
[b] Univ. Md. Department of Astronomy, College Park, MD 20742, USA

ABSTRACT

X-ray Phase Fresnel lenses (PFLs) can be considered as diffraction gratings with rotational (axial) symmetry and radially-varying pitch. The achromatic combinations of refractive and diffractive lenses that have been proposed for applications in X-ray and gamma-ray astronomy may then be regarded as grisms, again with variable pitch and axial symmetry. This way of looking at optics for very high angular resolution high-energy astronomy leads to the consideration of systems that bridge the gap between focusing and interferometry. X-ray diffractive Axicons and PFLs are shown to be limiting cases of a family of designs that are the X-ray equivalents of "Axilenses", offering different combinations of effective area and bandpass. It is shown that linear gratings can be used as diffractive alternatives to the grazing incidence mirror "periscopes" that have been investigated as beam combiners in an interferometer. The gratings form achromatic fringes in a process related to the Talbot effect. The results of simulations and of a laboratory demonstration-of-principle experiment are presented.

Keywords: X-ray imaging, Gamma-ray imaging, X-ray interferometry, Gamma-ray interferometry

1. INTRODUCTION

A Fresnel zone-plate (Fig. 1a) is a focussing device in which the amplitude of an incoming wave-front is modulated by alternating opaque and transparent zones in such a way that the only radiation that is transmitted has a relative phase in the range $\phi = \phi_0 - 90°$ to $\phi_0 + 90°$ when it arrives at the focal point. A small section of a zone-plate at radius r can be considered as a diffraction grating for which radiation incident normally and diffracted in the first order is directed towards the focal point, implying that $p(r)$, the pitch of the cyclic pattern at radius r, is given by

$$p(r) = \frac{f\lambda}{r}. \tag{1}$$

Looked at from this point of view, the low efficiency of a zone-plane results from the loss of energy by absorption (50%), and by diffraction into the zeroth order (25%), into negative orders (14.8%) and into orders $n > +1$ (4.7%), leaving just 10.1% in the $n = +1$ focus. Phase Fresnel lenses (PFLs, Fig. 1b) are a variation of zone-plates in which the *phase* of the transmitted radiation is modified so that all radiation arrives with exactly the same phase. In a loss-free PFL, all of the energy goes into the $n = +1$ focus. In the X-ray and gamma-ray regimes refractive indices are (slightly) less than unity, so the profiles is a stepped *concave* shape. Each part of the lens is now a phase grating blazed so that all of the energy goes into order $n = +1$.

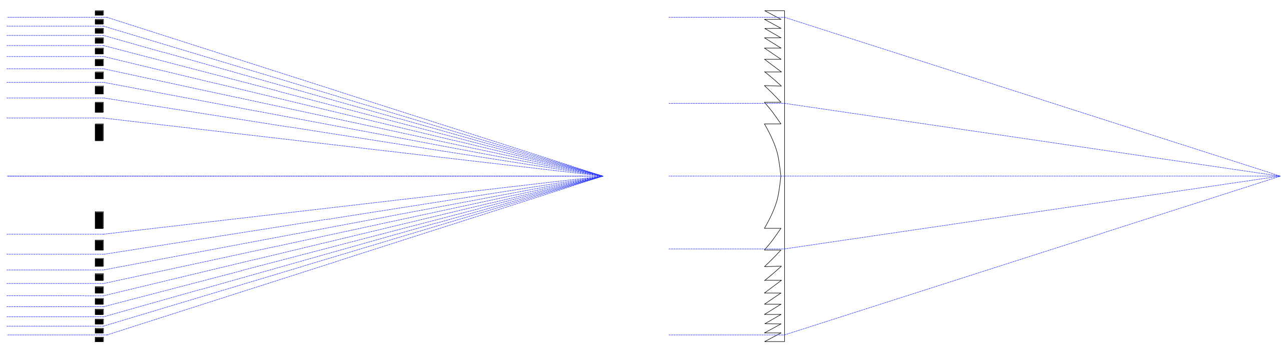
At X-ray and gamma-ray wavelengths PFLs can have low absorption losses, leading to near 100% efficiency, and can offer close to diffraction-limited angular resolution. For this reason they have been discussed as a means of obtaining the sub-micro-arcsecond necessary for some astrophysical objectives, particularly for imaging space-time in the environment of the super-massive black holes (SMBHs) at the centers of active galaxies.[1,2]

A major disadvantage of PFLs (or zone-plates) is that, for a given detector distance, the superb focussing is achieved only over a very narrow energy band. This problem can be alleviated by combining the PFL with a refractive lens in an 'achromatic' PFL combination though the bandwidth is still rather limited and as in most practical cases the refractive component has to be stepped to reduce absorption, the correction is only perfect at a limited range of wavelengths within the bandpass.[1,3–5]

At gamma-ray energies, a PFL (with or without achromatic correction) a few metres in diameter can achieve sub-micro-arcsecond resolution (the Rayleigh limit for a 5m diameter lens at 500 keV is 0.12 micro-arc-seconds).

Send correspondence to G.K.S: E-mail: skinner@milkyway.gsfc.nasa.gov

Optics for EUV, X-Ray, and Gamma-Ray Astronomy IV, edited by Stephen L. O'Dell,
Giovanni Pareschi, Proc. of SPIE Vol. 7437, 74370J · © 2009 SPIE
CCC code: 0277-786X/09/$18 · doi: 10.1117/12.826853



(a) (b)

Figure 1. (a) Focussing radiation with a zone plate (b) A corresponding Phase Fresnel Lens (PFL).

There are reasons, however why it would be preferable if possible to make measurements at lower energies – the photon flux is far greater, offering better statistics, and the Fe emission lines at 6–7 keV provide valuable information, particularly through their Doppler and gravitational shifts. For a given angular resolution, however, the scale of a lens needed is ~ 100 times larger.

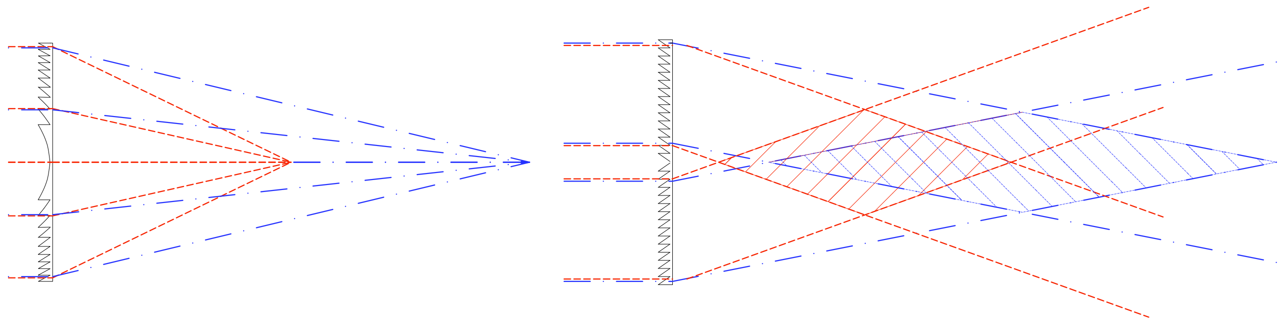
A 500 m diameter lens is not very practicable, nor is it necessary in order to collect adequate flux. It has been suggested that an effective area of 1000 cm^2 is the minimum necessary to make ultra-high angular resolution observations of SMBHs around the Fe lines,[6] though more would certainly be desirable. Thus one is led to consider unfilled aperture instrumentation in which a sparse array of small optics, distributed over an area several hundred meters across, concentrate flux towards an imaging detector. When the number of optical elements is not large the point source response function (PSF) has strong side-lobes and it is becomes convenient to describe such a system as a multi-beam interferometer producing a fringe pattern rather than as an imager.

We discuss here various approaches to the problem of ultra-high X-ray/gamma-ray astronomy based on diffractive optics and spanning the range between lenses and interferometers. First (§2) we describe variations on the PFL concept that are equivalent to an X-ray (or gamma-ray) Axicon or Axilens. §4 considers systems that are best regarded as interferometers rather than lenses, the limiting case being a simple two-beam interferometer using diffractive optics to combine the beams. All of these configurations offer a much larger bandpass than a PFL. Finally Axilenses are compared with 'achromatic' combinations of diffractive and refractive components (§3).

2. X-RAY AXICONS AND AXILENSES

The narrow-band nature of a simple PFL is a direct result of the fact that the angle through which radiation is diffracted by a grating is wavelength dependent – the very effect that makes diffraction gratings useful for spectroscopy. The $1/r$ dependence on radius of the pitch p is exactly what is needed to direct radiation of a specific design wavelength towards the focus, but it is in consequence wrong for all other wavelengths.

Consider now what would happen if the pitch did not vary. All radiation of a given wavelength will be turned through the same angle, θ, with $\theta = \lambda/p$, assuming θ is small (Fig. 2b). The action is equivalent to that of a conical lens for visible radiation. This is a form of Axicon,[7] an axisymmetric device which forms a line focus. Using a reflective analogue, Fujiwara[8] showed that such a system has a PSF that is similar to a J_0^2 Bessel function. For such a PSF the integrated flux within radius r is approximately proportional to r provided r is neither too small nor too large. Remarkably, for a *diffractive* Axicon the form and scale of the PSF are largely independent not just of image plane distance but also of wavelength. This is true for at distance $0 \ll v \ll v_{max}$ where v_{max} is the distance at which radiation passing through the periphery of the optic crosses the axis (Fig. 2).



(a) (b)

Figure 2. (a) The chromatic nature of focussing by a PFL. Short dashed lines : long wavelength. Dash-dot lines : short wavelength (b) Long and short wavelengths focussed by a diffractive Axicon. Line focii are produced along the axis within the corresponding shaded region, which extends to distance v_{max} along the axis.

As a possible optic for X-ray or gamma-ray astronomy, a diffractive Axicon has properties that are somewhat the reverse of those of a PFL. While a PFL can focus flux with very high efficiency (effective area close to geometric area), for a given detector position it does so only in an extremely narrow band around a particular wavelength. Other wavelengths are focussed, but the focii are spread along the axis. On the other hand an X-ray Axicon works over a wide band but the efficiency is much lower. In fact for devices of similar size the integral $\int A_{eff}(E)dE$ is very similar. Here $A_{eff}(E)$ is the effective area for flux collected within the first diffraction ring in a detector at fixed distance v and integration is over $E = hc/\lambda$.

The capability of either a conical lens or an equivalent diffractive Axicon to produce a line focus has been widely used. For a uniform input beam, the on-axis intensity varies along the focal line, increasing approximately linearly with distance. There has been considerable discussion in the literature of techniques for changing the intensity distribution along the line focus produced by Axicons, in particular for making it more uniform for a given input beam profile (for example for laser machining or metrology), but also for producing shorter line focii[9] or even arbitrary distributions[10]. Devices capable of producing uniform illumination along a line focus of limited length are sometimes referred to as Axilenses[11]. Sochacki et al.[12] and others have shown that a uniform line focus extending from v_1 to v_2 can be obtained with an optic of outer radius R having a phase function

$$\phi(r) = \frac{R^2\pi}{(v_2 - v_1)\lambda} ln\Big(v_1 + (v_2 - v_1)\frac{r^2}{R^2}\Big). \quad (2)$$

This phase shift can be achieved with a stepped profile having overall depth $t_{2\pi}$ and with period

$$p(r) = \frac{v_2\lambda}{r}\left[\frac{v_1}{v_2} + \left(1 - \frac{v_1}{v_2}\right)\frac{r^2}{R^2}\right]. \quad (3)$$

This is Eqn. 1 with a correction term in square brackets that becomes unity if $v_1 = v_2$.

Of interest here is what happens if a diffractive optic with $\phi(r)$ given by Eqn. 2 is used with a detector in a *fixed* image plane, but with polychromatic radiation. Perhaps not surprisingly, the response as a function of λ or E is flat-topped and of limited extent (Figs. 4 and 5). It provides a compromise between a PFL and an Axicon. The near-constancy of $\int A_{eff}(E)dE$ continues to apply. Although the PSF is not independent of wavelength, it changes comparatively little over the bandpass.

3. COMPARISON OF AXILENSES AND DIFFRACTIVE/REFRACTIVE 'ACHROMATIC' COMBINATIONS

Use of a diffractive/refractive achromat has been widely discussed as a way of broadening the bandpass of an X-ray PFL (e.g. refs 1,3–5). Because the refractive component would otherwise be so thick as to be impracticable,

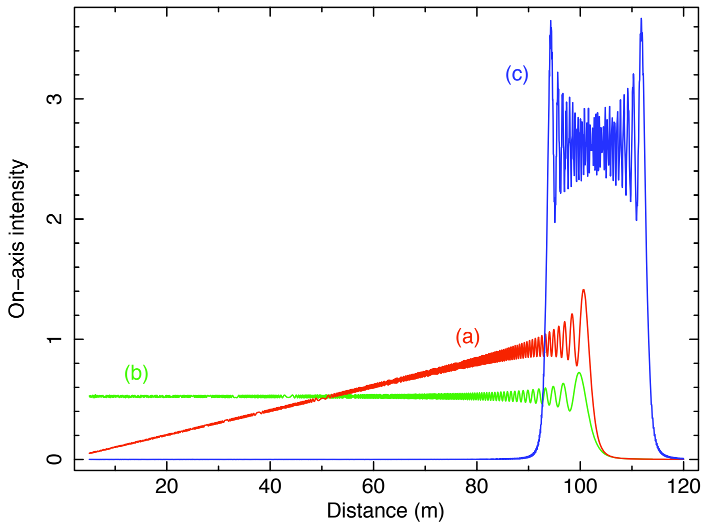


Figure 3. The on-axis response as a function of axial distance for (a) an Axicon with a maximum focal distance of 100 m (b) an Axilens covering the same range (c) an Axilens designed for the limited axial range v_1 to v_2 = 90–110 m. The results shown are for a 1 cm diameter optic operating at 6.5 keV.

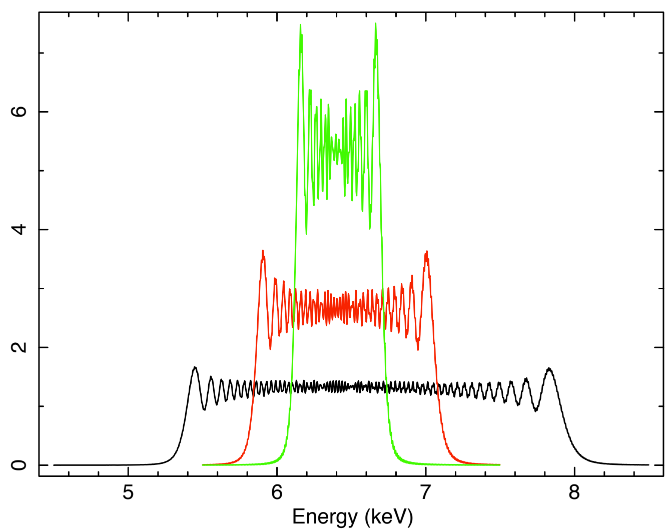


Figure 4. The on-axis response as a function of photon energy at a fixed distance of 100 m for Axilenses similar to that in Fig. 3(c) with three different combinations of v_1, v_2. The ratios v_2/v_1 are 1.1, 1.22 and 1.55 for the curves with the highest to the lowest peaks.

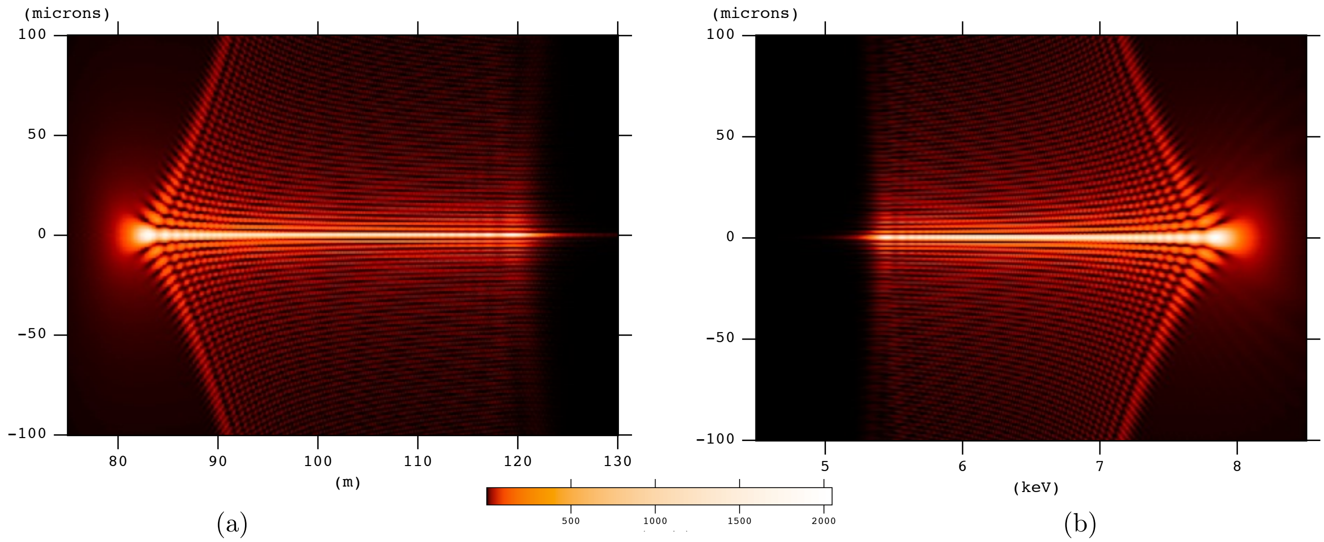


Figure 5. (a) The response of an Axilens designed with v_1=80 m, v_2=120 m, as a function of image plane distance (horizontal axis) and off-axis distance (vertical axis) at a fixed photon energy (6.5 keV) (b) A corresponding plot at fixed image plane distance (100 m) as a function of photon energy and off-axis distance. In each case the on-axis intensity is approximately constant within the operating range. Over much of the range the PSF (a cross-section though this plot along a vertical line) changes little. The 'flaring' towards the short distance, or high energy, limit occurs because the PSF broadens when only a small region at the center of the optic is effective.

as well as absorbing virtually all the radiation, it has to be stepped, leading to an on-response that is close to ideal at a comb of wavelengths, but that drops to be close to zero at intermediate wavelengths, where the energy is diverted into sidelobes in the PSF.[3] The improvement in bandwidth is less than that possible with an Axicon or with an Axilens with large v_2/v_1, but the integrated response $\int A_{eff}(E)dE$ can be much larger. It is actually better than for a simple PFL, instead of being similar. The main difference is that the on-axis response is concentrated into a series of peaks.

In principle it may be possible to combine both the Axilens approach to widening the bandpass with a refractive correcor, but at present a useful combination has not been identified.

4. PARTIALLY FILLED APERTURES

As mentioned in §1, to obtain micro-arcsecond angular resolution at 6–7 keV would require an optic several hundred meters in diameter, which is far beyond present technological limits. There has been a proposal, MAXIM,[13] to achieve such resolution in this band, but with a multibeam interferometer. The MAXIM concept uses mirror assemblies ('periscopes') on a number of spacecraft separated by distances d up to 1 km to combine beams together on a detector on another spacecraft, a large distance (e.g. $f=$ 20000 km) away.

It is interesting to consider whether the technologies under consideration here could be applied in such a configuration. Each MAXIM periscope uses successive reflections from four high-precision grazing incidence mirrors to divert an incoming beam by a small angle, $d/2f$. Regarded as a partially-filled-aperture focussing system, within the small sub-aperture corresponding to a each periscope the deviation is everywhere by the same angle in the same direction, so a parallel input beam leaves the periscope as a parallel beam. At a given wavelength, the equivalent using diffractive optics would be to divert the radiation with a simple fixed-pitch linear phase grating. The deviation would, however be wavelength dependent and to accommodate a range of wavelengths the diffracting element would have to be extended as indicated in Fig. 8.

This arrangement has the interesting property that the fringes, unlike those obtained using mirrors to combine the beams, are achromatic. This is because at longer wavelengths the beams converge at a proportionately steeper

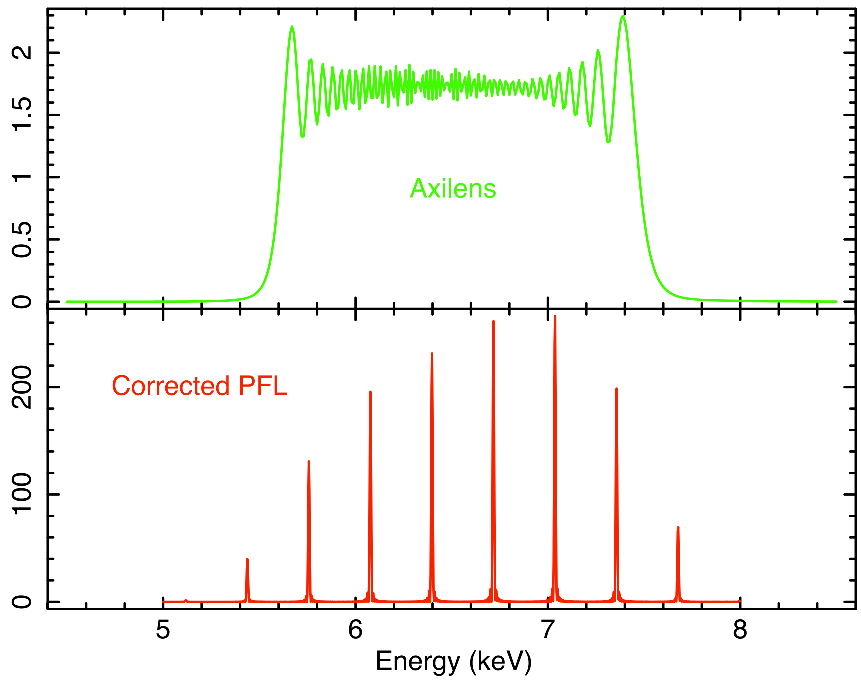


Figure 6. Comparison of the on-axis response of an example Axilens (upper plot) with that of a similar size diffractive/refractive achromatic PFL (lower plot). Each optic is assumed to be made from polycarbonate and to have a ratio of detector distance to diameter (the f-ratio defined for a classical lens) of 10^4. For the Axilens v_1 and v_2 have been chosen to give a bandpass similar to that of the achromat.

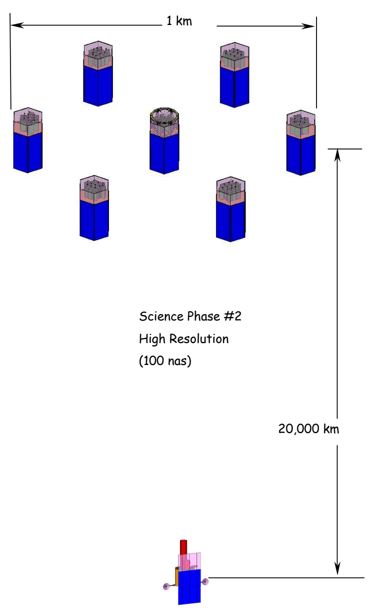


Figure 7. One of the concepts for the MAXIM micro-arc-second imager mission (from Gendreau et al.[14]).

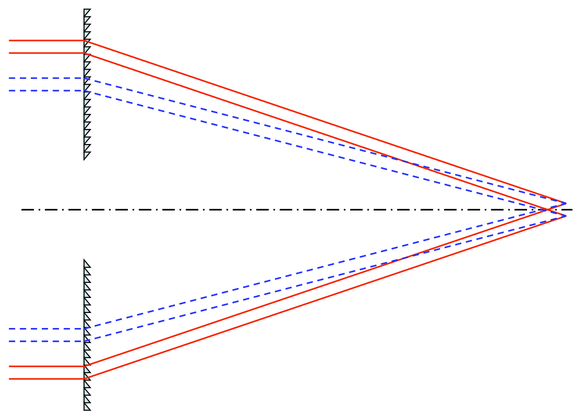


Figure 8. The use of diffractive beam combiners in a 2-beam interferometer. Each diffractor is effectively a blazed transmission grating. So that different wavelengths, such as the two shown, converge on the same spot, the diffractors must be extended radially. Not to scale – in practice the grating pitch would be much finer than indicated and the point of convergence more distant.

angle. In fact they have a pitch that is just half that of the gratings (their production is in many ways analogous to the Talbot effect – the formation of a self-image behind a periodic structure).

Another important attribute of the formation of fringes using diffractive beam combiners in transmission is that they are remarkably insensitive to tipping or tilting of the diffractors. An analysis of the permissible deviations in positions and angular alignment[15] shows that the only requirement not readily met by standard technologies is the need to control the radial position of the diffractors, relative to the line of sight, to within a small fraction of the pitch, p. In fact for a large range of alignments fringes will be formed throughout the large volume of space in which the beams cross; **knowledge** of the radial positions is needed if the phase of the fringes is to be used and **control** of them is needed to obtain the full benefit of constructive interference between multiple beams.

A two-beam interferometer as shown in Fig. 8 is effectively a 1-dimensional form of diffractive Axicon, with gratings having linear rulings in place of circular ones. In fact if the rulings are made arcs of circles centered on the array axis, instead of straight lines, a considerable degree of concentration of flux is possible. The instrument becomes increasing like a partially filled aperture focussing optic.

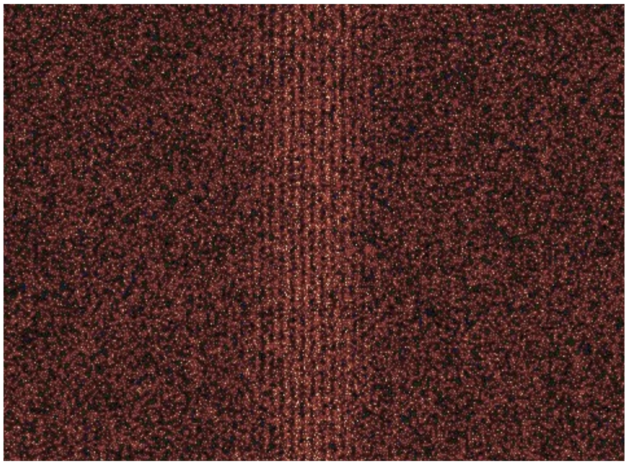
4.1 A Laboratory Demonstration

We have demonstrated the operation of a simple 2-beam X-ray interferometer with diffractive beam combiners, such as that illustrated in Fig. 8, using the 600 m X-ray interferometry testbed at NASA-GSFC*. The fringes seen in Fig. 9 were obtained with X-rays from a Copper target X-ray tube, containing a combination of the K_α, K_β lines and continuum radiation. Fringes were also obtained in the Tungsten L lines. The diffractors were simply plastic films profiled to have a sawtooth cross-section with period 55 microns, a material that is mass produced for quite different purposes (improving the visibility of displays) by 3M Corp. Because of the short source distance (146 m) compared with the distance of the detector plane (452 m), the fringes are magnified. Selecting individual lines demonstrates that they are indeed achromatic (Fig. 10). For more experimental details see Ref. 15

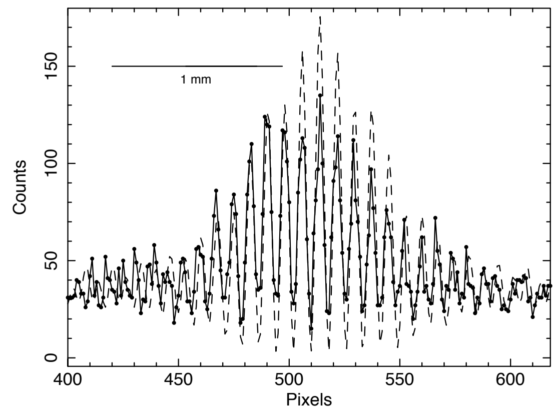
5. MULTI-BEAM INTERFEROMETERS USING DIFFRACTIVE COMBINERS

The analogy of the MAXIM configuration of Fig. 7 but using diffractive beam combiners would be as shown in Fig. 11. The beam combiners are shown all at the same radii from the axis because with refractive beam

*http://lheawww.gsfc.nasa.gov/~kcg/beamline/home.html



(a)



(b)

Figure 9. (a) Fringes obtained with diffractive beam combiners as described in the text. The events recorded are a mixture of Copper Kα (8.04 keV), Copper Kβ (8.90 keV) and continuum radiation. (b) A projection onto the horizontal axis of the data in (a). The dashed line show the result of a simulation.

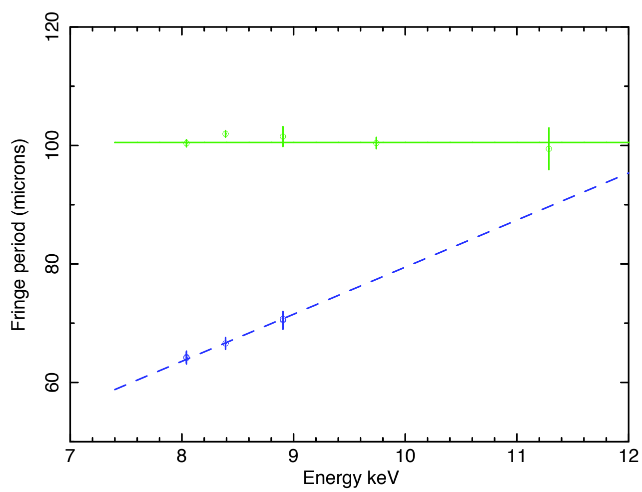


Figure 10. Upper (continuous) line: the period of fringes obtained with diffractive beam combiners as in Fig. 9, plotted as a function of the energy of the X-ray line (Cu $K_{\alpha,\beta}$, W $K_{\alpha,\beta}$). Lower (dashed) line: the period of similar fringes with refractive (prism) beam combiners for comparison (Cu $K_{\alpha,\beta}$). The lines show the predictions, data points measurements.

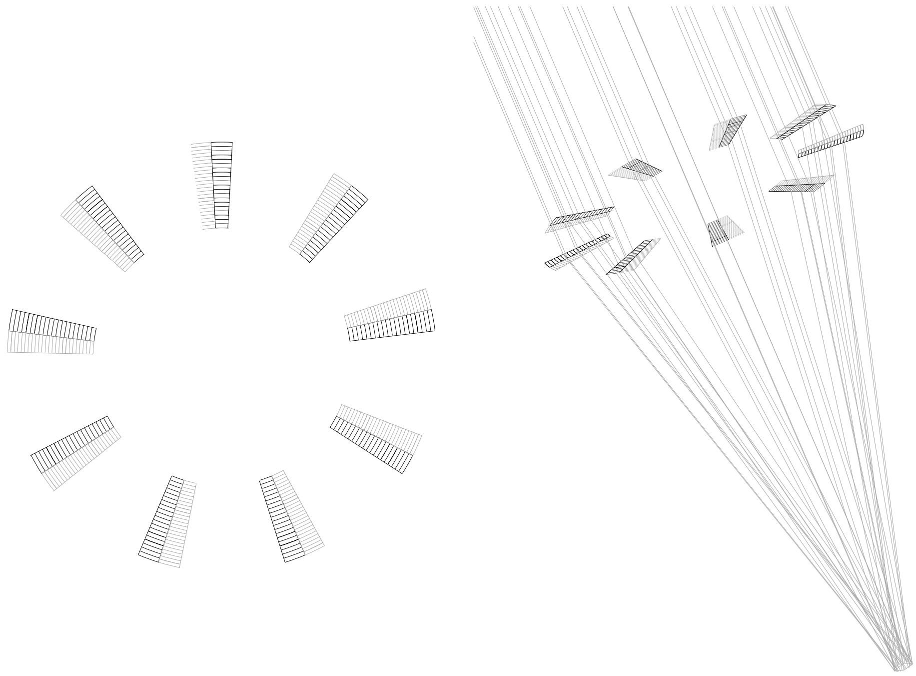


Figure 11. Use of diffractive beam combiners in a multi-beam configuration. Each unit is shown consisting of 2 diffractors, side-by-side, designed for different energy bands and concentrating flux on the same detector.

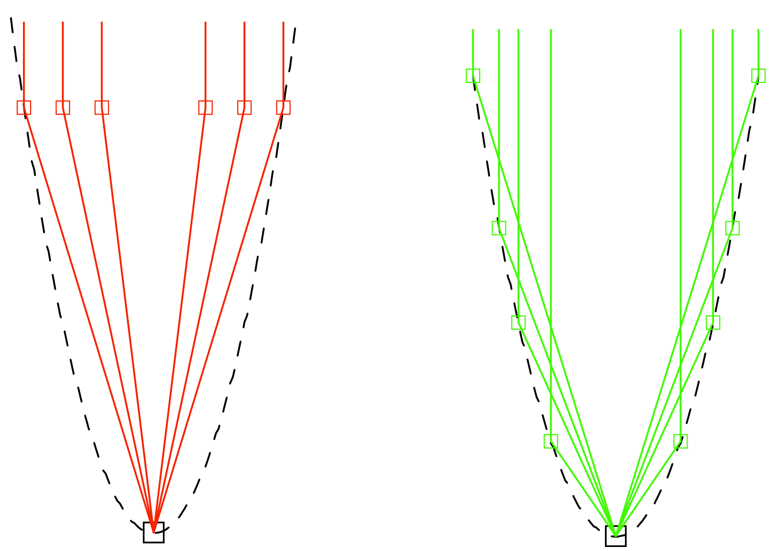


Figure 12. Left: If beam combiners lie on a plane perpendicular to the line of sight (as in Fig. 11) but at different radii, the path lengths are not equal unless compensated by a delay within the combiners. This is possible with mirror periscopes, but not with diffractive combiners. Right: To add coherently beams from diffractive beam combiners at different off-axis distances they would have to be distributed in three dimensions, over the surface of a paraboloid.

combiners it is not possible to compensate for the differences in path length that would otherwise occur even for on-axis radiation. This is unlike the situation with reflective periscopes where compensation is possible by adjusting the inter-mirror separation. The alternative would be to not limit the diffractor positions to a plane, but distribute them over a paraboloidal surface (Fig. 12).

6. CONCLUSIONS

It has been shown that the $1/r$ dependence of pitch on radius that is characteristic of a PFL or zone plate is a special case of a family of designs that extend at the other extreme to the 'X-ray Axicon', with constant pitch. Intermediate solutions in which the pitch varies in accordance with Eqn. 3 offer larger bandwidth than a PFL, though at the expense of reduced peak throughput. The integrated response is similar to that of a simple PFL, though inferior to that of a PFL combined with a stepped refractive component in an achromatic combination. In the case of the example shown in Fig. 6 the difference amounts to a factor $\sim$5. On the other hand the response of an Axilens as a function of energy is closer to a flat-topped function than the multi-peaked response of a stepped achromat.

It has also been demonstrated that related diffractive optics provide a simple way and robust way of making beam combiners for an interferometer that are very tolerant of misalignments. Although obtaining an extended bandwidth requires a diffractor of relatively large radial extent ($\Delta r/r \sim \Delta E/E$), the simplicity of the optics, their lax alignment constraints and the achromatic characteristic of the fringes mean that such devices may provide an attractive alternative to higher technology solutions.

ACKNOWLEDGMENTS

The author thanks J. Krizmanic for helpful discussions and for help with the experimental work and they are both grateful to K. Gendreau, Z. Arzoumanian and the team responsible for the development of the GSFC 600 m interferometry test bed. 3M Optical Service Division and Microsharp Corporation Ltd., U.K. provided film samples.

REFERENCES

[1] Skinner, G. K., "Diffractive/refractive optics for high energy astronomy. I. Gamma-ray phase Fresnel lenses," *Astron. & Astrophys.* **375**, 691–700 (Aug. 2001).

[2] Gorenstein, P., "Role of diffractive and refractive optics in x-ray astronomy," *SPIE Conference Series* **5168**, 411–419 (Feb. 2004).

[3] Skinner, G. K., "Design and Imaging Performance of Achromatic Diffractive-Refractive X-Ray and Gamma-Ray Fresnel Lenses," *Appl. Opt.* **43**, 4845–4853 (Sept. 2004).

[4] Gorenstein, P., Phillips, J. D., and Reasenberg, R. D., "Refractive/diffractive telescope with very high angular resolution for X-ray astronomy," *SPIE Conference Series* **5900**, 369–376 (Aug. 2005).

[5] Braig, C. and Predehl, P., "Efficient Fresnel x-ray optics made simple," *Appl. Opt.* **46**, 2586–2599 (May 2007).

[6] Gendreau, K. et al., "MAXIM micro arcsecond X-ray Imaging mission - a proposal in response to Call for Mission Concepts NRA 03-OSS-01-VM," tech. rep., NASA-GSFC (2003).

[7] McLeod, J. H., "The axicon: A new type of optical element," *Journal of the Optical Society of America (1917-1983)* **44**, 592 (Aug. 1954).

[8] Fujiwara, S., "Optical properties of conic surfaces. (1) reflecting cone," *J. Opt. Soc. Am.* **52**(3), 287–291 (1962).

[9] Sochacki, J., Klodziejczyk, A., Jaroszewicz, Z., and Bara, S., "Nonparaxial design of generalized axicons," *Appl. Opt.* **31**, 5326–5330 (Sept. 1992).

[10] Cao, Q. and Chi, S., "Axially symmetric on-axis flat-top beam," *Journal of the Optical Society of America A* **17**, 447–455 (Mar. 2000).

[11] Davidson, N., Friesem, A. A., and Hasman, E., "Holographic axilens: high resolution and long focal depth," *Optics Letters* **16**, 523–525 (Apr. 1991).

[12] Sochacki, J., Bara, S., Jaroszewicz, Z., and Kolodziejczyk, A., "Phase retardation of the uniform-intensity axilens," *Optics Letters* **17**, 7–9 (Jan. 1992).

[13] Cash, W. C., "Maxim: micro-arcsecond x-ray imaging mission," *SPIE Conference Series* **4852**, 196–209 (Feb. 2003).

[14] Gendreau, K. C., Cash, W. C., Shipley, A. F., and White, N., "MAXIM Pathfinder x-ray interferometry mission," *SPIE Conference Series* **4851**, 353–364 (Mar. 2003).

[15] Skinner, G. K. and Krizmanic, J. F., "X-ray interferometry with transmissive beam combiners for ultra-high angular resolution astronomy," *To be published in Experimental Astronomy (arXiv 0907.5365)* (2009).